1. a). Given three vectors

$$\vec{a}=[a_1,a_2,a_3],\vec{b}=[b_1,b_2,b_3],\vec{c}=[c_1,c_2,c_3],$$

compute $\vec{a}\cdot(\vec{b}\times\vec{c})$ and **explain** the geometrical interpretation of the scalar triple product. (7%)

b). Given the curve: $\vec{r}(t)=t\,\vec{i}+\cosh t\,\vec{j}$, $t\in[0,1]$, **compute** the length of $\vec{r}(t)$ from $t=0$ to $t=1$, i.e., $\int_0^1\sqrt{\vec{r}'(t)\cdot\vec{r}'(t)}dt$, and **explain** the geometrical meaning of $\vec{r}'(t)$. (7%)

c). Find the directional derivative of $f(x,y,z)=xyz$ at $P(-1,1,3)$ in the direction of $\vec{a}=\vec{i}-2\vec{j}+2\vec{k}$. (6%)

2. Consider the matrix $A=\begin{bmatrix}1&0&0&0\\1&-1&0&0\\0&0&1&1\\0&0&0&-1\end{bmatrix}$

a). What is Cayley-Hamilton theorem ? Give a simple example to demonstrate it. (5%)

b). $A^{30}=?$ (5%)

c). What is the determinant of A^{30} ? (5%)

d). How many linearly independent eigenvectors does matrix A have ? (5%)

3. Evaluate the following integrals:

$$I_1=\int_0^\infty\frac{\cos ax}{x^2+1}dx \quad\text{and}\quad I_2=\int_0^\infty\frac{\sin ax}{x^2+1}dx \quad (a>0)$$

by using the *Residue Theorem* in the complex variables theory. (20%)

4. Use the Fourier series method to solve the problem: (20%)

$$u_t=3u_{xx} \qquad 0<x<2, t>0$$

with

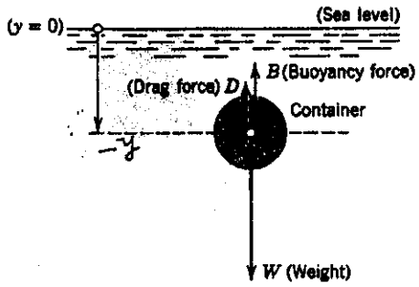
$$u(0,t)=u(2,t)=0, \qquad t>0$$
$$u(x,0)=2[1-\cos(\pi x)], \qquad 0<x<2$$

(背面仍有題目,請繼續作答)

5. The Newton second law of motion: the equivalent force of mass times acceleration is equal to resulting force of weight, buoyant force, and drag force (say, $ma = \sum \text{force}$).

a). A spherical ball of weight 1 [kg] is immerged in water and moving upward. If the buoyancy force is 1 [nt], and the drag force is $D = \alpha \cdot V$, where $\alpha = -0.1$ [nt·s/m] and V is the velocity. The initial velocity is $V_0 = 10$ [m/s], and initial location is at $y(0) = -200$ [m]. The gravitational acceleration is 9.8 [m/s^2]. Find the velocity distribution with respect to time. (10%)

Note: [nt] = [Newton], 1 [nt] = 1 [kg·m/s^2].



b). Consider the simplified mass-spring system, with mass $m = 1$ [kg], spring constant $k =$ 20 [nt/m], the damping constant is $c =$ 4 [kg/s], input force is $r(t) = 0.1 \times \cos(4t)$ [nt]. If the initial location of mass is at $y = 0$ [m], find the location of the mass with respect to time. (10%)