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1) 20%

Consider a two-dimensional flow field,

a) Please give the definitions of streamline, pathline, and steakline.

b) Let the velocity (u, v) be the function of x, y, and t (t : time):

$u = x(1+2t)$, $v = y$.

Find the equation of:

i) The streamline through the point (1,1) at $t = 0$,

ii) The pathline for a particle released at the point (1,1) at $t = 0$.

2) 20%

Consider the steady flow field between two infinite vertical walls in the figure below. The wall at $x=0$ moves at a steady velocity $V_0\hat{j}$, and the wall at $x=L$ moves at a steady velocity $2V_0\hat{j}$, where $\hat{j}$ is the unit vector in y-direction (vertical).

(a) Assume that the flow velocity is in y-direction only. That is, $\vec{v} = 0\hat{i} + V(x)\hat{j}$, where $\hat{i}$ is the unit vector in x-direction (horizontal). Verify whether this flow field is incompressible or compressible.

(b) The two-dimensional momentum equation in vector notation is

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + \mu\nabla^2\vec{v}$$

where ρ is density, P is pressure, μ is viscosity, $\vec{\nabla}$ is the gradient operator, and ∇^2 is the Laplacian operator. For the flow field in (a), simplify the above equation to obtain the governing equations for $V(x)$ and P.

(c) Assume that P and μ are constant everywhere. Solve for $V(x)$ with appropriate boundary conditions.

(d) What is the viscous shear stress acting on the right wall at $x=L$? Is this viscous stress in $+y$ or $-y$ direction?

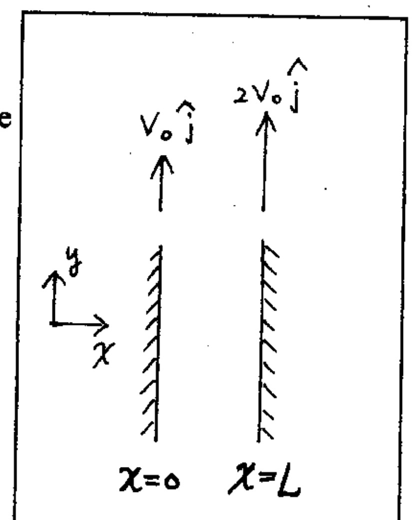

Fig 2

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3) 20% For the incompressible boundary layer flow, derive the steady-flow two-dimensional momentum-integral relation (including wall suction or blowing), which is expressed by the variables $\tau_w, \theta, \delta^*, U, \rho, and\ V_w$, by applying conservation laws to the control volume shown.

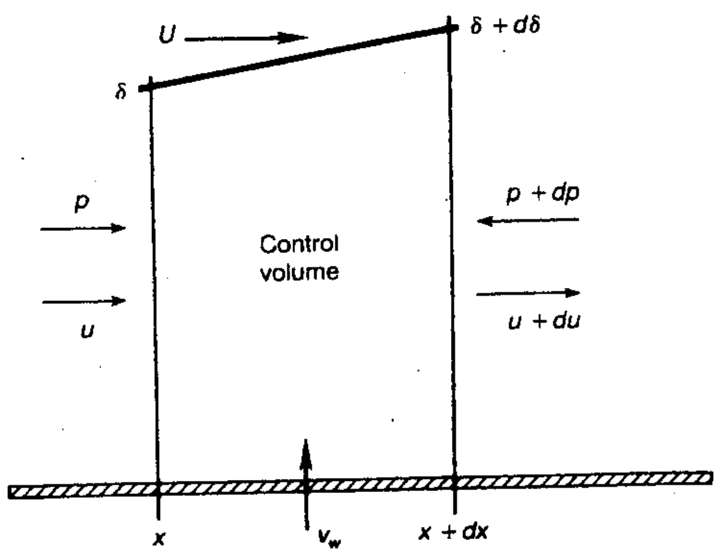

Fig 3

Hint: (1) If U is constant and V_w is equal to zero, the momentum-integral relation can be expressed as follows:

$$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx}$$

, where τ_w is the wall shear stress and θ is the momentum thickness.

(2) δ : boundary-layer thickness

δ^* : displacement thickness

4) 20%

Consider a cylindrical container, partially filled with liquid as shown in Fig. 1, is rotated at a constant angular speed ω about its axis. Determine the shape of the free surface. That is, show that

$$z = h_1 + (\omega r)^2 / 2g .$$

Hint: A fluid particle is acted by the forces due to pressure and gravity. Thus you need to write down the equation of motion for the fluid particle. Note that the gradient operator in the cylindrical coordinate is $\nabla = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{\partial}{r\partial\theta} + \vec{k}\frac{\partial}{\partial z}$, and the pressure p is a function of coordinates r and θ.

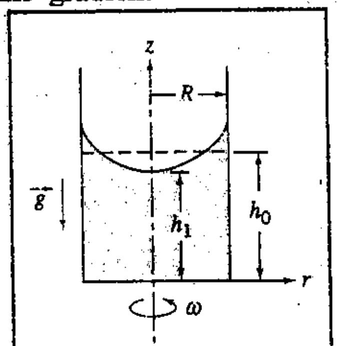

Fig 4.

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5) 20%

Consider a huge water tank with a special arrangement of exit as shown where the distance from the water level to the center line of exit is 10 m. Assume the exit has a properly rounded inlet (with circular shape and exit diameter $D =$ 1 cm), the entrance loss can be estimated as $h_E = 0.03V^2/2g$, where V is the mean exit velocity and $g = 9.8\,\mathbf{m}/\text{sec}^2$. A straight expansion pipe of length $L =$ 10 cm with inlet diameter $D_{in} =$ 1cm and the exit diameter $D_{out} = 3\text{cm}$ is connected to the exit. The loss of the expansion pipe can be estimated by $h_p = 0.02(L/\overline{D})\,V_{in}^2/2g$, where $\overline{D} = 0.5(D_{in} + D_{out})$.

(a) Neglect the viscous loss, estimate the mean volume flow rate through the exit with and without the expanded connecting pipe. (10%)

(b) Consider the viscous loss, estimate the mean volume rate through the exit with and without the expansion connecting pipe. (10%)

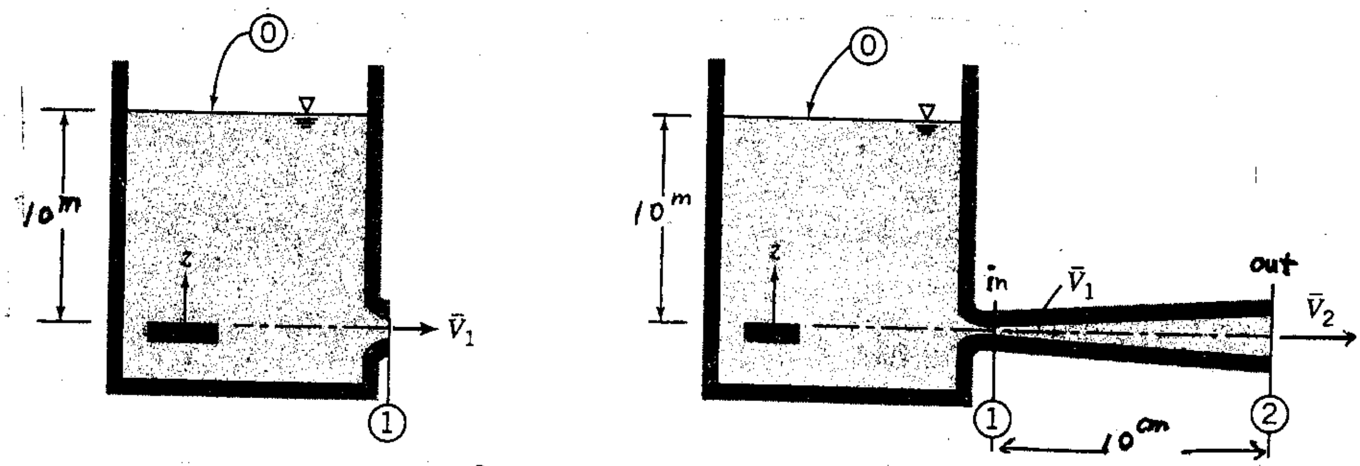

Fig 5.