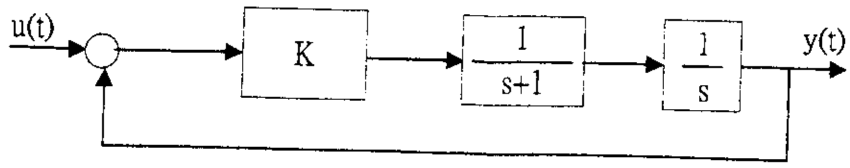
本試題是否可以使用計算機： □可使用 ， □不可使用 （請命題老師勾選）

1 (a) (10%) Consider the following 2^{nd} order system.



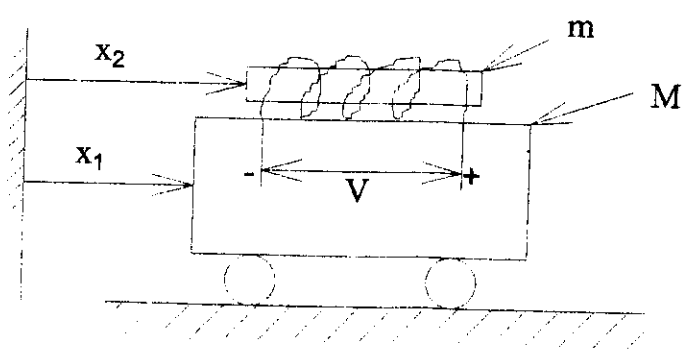
Find the range of the proportional feedback gain K so that the system is stable.

(b) (10%) Suppose the output of the block $\frac{1}{s+1}$ should be bounded by 10, and this system is excited by a sinusoidal input of the following form

$$u(t) = A(\omega)\sin\omega t, \quad -\infty < t < \infty .$$

Find the range of the amplitude A, as a function of ω and K, so that the system won't saturate.

2 Consider the system as shown below, where mass m is a magnetic bar moving frictionlessly in a massless solenoid mounted on mass M. Let x_1 and x_2 be the absolute positions of M and m, respectively. A linear force F is applied to m in the positive x_2 direction if a positive voltage V is applied to the solenoid, and $F=\mu V$, where μ is a constant. For simplicity, let $\mu=1$.



(a) (5%) Derive the governing equations and the state space realization, $\dot{x} = Ax + BV$, of the system.

(b) (5%) Is the equilibrium states $(x_1, \dot{x}_1) = (x_2, \dot{x}_2) = 0$ stable when V= 0? Why?

(c) (5%) If the initial condition of the system is $(x_1, \dot{x}_1) = (x_2, \dot{x}_2) = 0$, can you find a position control law to move the system to any specified position x_r so that $(x_1, x_2) = x_r$, and $(\dot{x}_1, \dot{x}_2) = 0$. If yes, derive the control law. If no, why?

(d) (5%) Is the system completely state controllable? Explain your answer from the dynamic point of view.

(背面仍有題目,請繼續作答)

3 Answer the following questions:

(a) (5%) For a stable system $M(s)$ that is under a sinusoidal input $r(t) = A\sin\omega t$, $A \in R$, what is the steady state response of the system ?

(b) (5%) What is a low-pass filter ? What kind of transfer functions act as a low-pass filter ?

4 Consider the LTI SISO plant with the following state-space realization

$\dot{x}(t) = Ax(t) + bu(t)$, $y(t) = cx(t)$

where $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $c = \begin{bmatrix} 1 & 0 \end{bmatrix}$

(a) (5%) What is the transfer function $G_p(s) = \dfrac{Y(s)}{U(s)}$ of the plant ?

(b) (5%) Check the controllability and observability of the plant ?

(c) (5%) If we apply a linear state feedback control law $u(t) = -Kx(t)$ to assign the poles at $-2, -5$. What is the value of the gain K.

(d) (5%) Suppose that the states $x(t)$ are not accessible, we need to design an observer to estimate the states before you use the state feedback to assign the poles. The observer is supposed to be

$\dot{\hat{x}}(t) = A\hat{x}(t) + bu(t) + L(y(t) - c\hat{x}(t))$, where L is the observer gain and

$\hat{x}(t)$ is the estimate of the actual state $x(t)$. Define the error in the estimation $\tilde{x}(t) = x(t) - \hat{x}(t)$. To see the error $\tilde{x}(t)$ evolving with time, derive the error dynamic equation for $\tilde{x}(t)$.

(e) (5%) In (4), if we assign the poles (or eigenvalues) of the observer at $-8, -9$, find the observer gain L.

(f) (5%) If we now apply the estimated state $\hat{x}(t)$ to construct the state feedback control law $u(t) = -K\hat{x}(t)$ and again assign the poles at $-2, -5$. What is the value of the gain K.

5 (10%) Satellite Problem:

Model the earth and the satellite as particles. Consider the earth fixed in an initial frame. The normalized equations are (from Lagrange):

$$\begin{cases} \ddot{r} = r\dot{\theta}^2 - \dfrac{k}{r^2} + u_1 \\ \ddot{\theta} = -2\dfrac{\dot{\theta}\dot{r}}{r} + \dfrac{1}{r}u_2 \end{cases}$$

where u_1, u_2 represent the radial and tangential forces. The orbit is $r(t) = \rho,\ \theta(t) = \omega t$ and Kepler insists that $\rho^3\omega^2 = k$. Linearize the equations about this orbit.

6 (10%)

Let A be a Hermitian matrix with distinct eigenvalues λ_i and $\bar{v}_i$ the corresponding eigenvectors. Then the differential equation

$\dot{x} = Ax,\ x(t_0) = \bar{v}_i$ always has a solution in R^n which lies in a straight line.

True or false? Explain briefly.