1. Vector Analysis:

   (a) Let $F(x, y, z) = (y, 2xz, ze^x)$, compute $\operatorname{div} F$ and $\operatorname{curl} F$. (7%)

   (b) Let $f(x, y, z) = x^2 y \cos(yz)$, compute $\operatorname{grad} f$. (6%)

   (c) Evaluate the line integral   (7%)

   $$\int_C x\,dx - xy\,dy$$

   If C is given by $x = t^2, y = -t;$　　$1 \le t \le 2.$

2. Find the general solution of the following differential equations:

   (a) $y^2 dx + (2xy - x^4) dy = 0$ (6%)

   (b) $y'' - 2y' + y = e^x + x$ (7%)

   (c) $\mathbf{x}' = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -8 \\ 3 \end{bmatrix}$ (7%)

3. Given a $n \times n$ matrix A we wish to find a $n \times 1$ vector X so that $AX = Z$ for a given $n \times 1$ vector Z. List all conditions on A and Z in order that a solution may exist. (20%)

4. (20 points)

   a). Let

   $$f(z) = \frac{z^{1/2}}{(z+1)(z-2)^2},$$

   please locate all the singularities and determine their nature.

   b). Evaluate the integral

   $$\frac{1}{2\pi i} \int_{|z|=3} \frac{dz}{(z^2+1)(z-2)^2(z-4)}.$$

（背面仍有題目,請繼續作答）

5. With the help of the schematic diagram, solve the following two-dimensional boundary value problem within the square in term of the method of separation variable.
(20%)

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

$U = 1,\ 1 < x < 3,\ y = 1$

$U = 0,\ 1 < x < 3,\ y = 3$

$U = 0,\ x = 1,\ 1 < y < 3$

$U = 0,\ x = 3,\ 1 < y < 3$