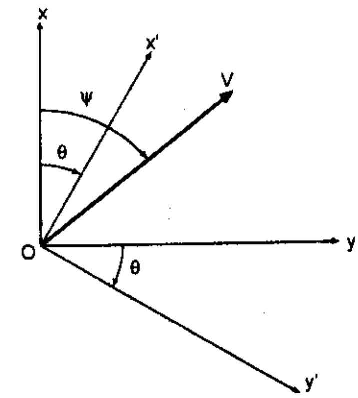
系所組別： 航空太空工程學系丙組

考試科目： 動力學

考試日期：0307，節次：2

※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

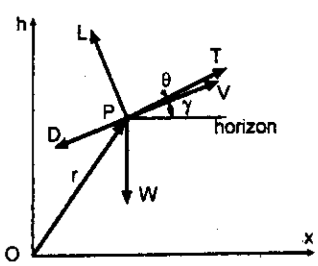
Problem 1. As shown in the following figure, the frame $Ox'y'$ is obtained by rotating the frame



Oxy clockwisely an angle θ about the point O. The heading angle ψ is defined as an angle rotating clockwisely from the x-axis to the velocity $\mathbf{V}$. With given θ, V, and ψ, determine the components of the velocity (a) in frame Oxy and (b) in frame $Ox'y'$. Let $\mathbf{ij}$ and $\mathbf{i'j'}$ be the sets of unit vectors in the frames Oxy and $Ox'y'$, respectively. If $\mathbf{V} = 2\,\mathbf{i} + 3\,\mathbf{j}$ and $\theta = 30°$, express $\mathbf{V}$ in the components of the $Ox'y'$ frame, i.e., if $\mathbf{V} = a\,\mathbf{i}' + b\,\mathbf{j}'$, then what are a and b? (25%)

Note: The bold face represents vector while the normal style represents scalar.

Problem 2. A particle with mass m is located at point P as shown in the following figure. Let



Oxh be a fixed frame, with x and h being the horizontal range and the altitude, respectively. The position vector of the particle is $\mathbf{r}$ and its velocity vector $\mathbf{V}$. The angle between the velocity and the horizon is γ, which is called the flight path angle. The particle is acted by the thrust $\mathbf{T}$, the lift $\mathbf{L}$, the drag $\mathbf{D}$, and the gravity $\mathbf{W}$ as shown. According to the definition, the drag is always parallel to the velocity while the lift is always vertical. With the above information, determine

$$\frac{dx}{dt},\quad \frac{dh}{dt},\quad \frac{dV}{dt},\quad \text{and} \quad \frac{d\gamma}{dt}$$

in terms of m, V, γ, T, θ, L, and D, where t is the time. Let the gravity acceleration be g. (25%)

Note: The bold face represents vector while the normal style represents scalar.

(背面仍有題目,請繼續作答)

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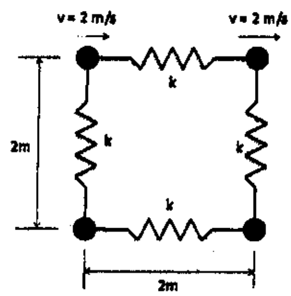
Problem 3. Four 0.1 kg particles in space are attached together with springs as shown. Initially the springs are unstretched and the two top particles are given a velocity of 2 m/sec to the right as shown.



(a). Define some reference frame and state what the future velocity of the mass center will be in your frame, ignore gravity.

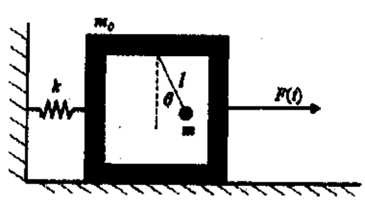
(b). Compute H (angular momentum about the mass center) and express it in your frame from (a) at $t = 0$ sec and $t = 0.5$ sec, ignore gravity.

(c). Compute the total energy of the system at $t = 0$ sec and $t = 0.5$ sec ignoring any effects of gravity.

(d). Now include a gravitational force acting vertically downward on the page and state what the velocity of the mass center will be after 0.5 sec.

(25%)

Problem 4. A box of mass m_0 supports a simple pendulum of mass m and length l. A spring of stiffness k forms a horizontal mass-spring system with the box which can slide without friction on a fixed horizontal surface. An external force $F(t)$ is applied to the box as shown.



(a). How many degrees of freedom does this system have?

(b). Find the equations of motion of the system using Newtons Second Law. Designate reference frames where necessary and use good notation.

(c). Find the equations of motion of the system using Lagranges equations. You should be able to reuse your velocity calculations from part (b).

(25%)