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Problem 1 (20%)

Solve the initial value problem

$$y'' + 2y' + y = e^{-t}, \quad y(0) = -1, \quad y'(0) = 1,$$

(1). by the method of undetermined coefficients. (10%)

(2). by the Laplace transform. (10%)

Problem 2 (20%)

(1). Let $\bar{r}(t)$ denote a three-dimensional vector as function of t. Also, let $\dot{\bar{r}}$ and $\ddot{\bar{r}}$ denote $\frac{d\bar{r}}{dt}$ and $\frac{d^2\bar{r}}{dt^2}$, respectively.

(a). Prove that $\bar{r} \bullet \dot{\bar{r}} = r\dot{r}$, where r is a scalar denoting the magnitude of $\bar{r}$. (5%)

(b).If $\bar{r}$ satisfies the differential equation, $\bar{r} \times \ddot{\bar{r}} = \bar{0}$, prove that this differential equation can be integrated to $\bar{r} \times \dot{\bar{r}} = \bar{C}$, where $\bar{C}$ is a constant vector. (5%)

(2). It is noted that if a position vector $\bar{r}$ of a point can be denoted as a function of two parameters, α and β, i.e., $\bar{r} = \bar{r}(\alpha, \beta)$, then the increment of area due to the increments of two parameters can be denoted as

$$dA = \left| \frac{\partial \bar{r}}{\partial \alpha} d\alpha \times \frac{\partial \bar{r}}{\partial \beta} d\beta \right|$$

Now, if $\bar{r} = \bar{r}(r, \theta)$, where r is the magnitude of $\bar{r}$ and θ is the angle between $\bar{r}$ and a fixed reference line, prove that $dA = r\,dr\,d\theta$ for this case by using the above equation directly. (10%)

Problem 3 (20%)

(1). Show that eigenvectors associated with distinct eigenvalues are orthogonal for a real symmetric matrix. (5%)

(2). (a). Let $Q = 17x_1^2 - 30x_1x_2 + 17x_2^2$. Find a real symmetric matrix A such that

$Q = [x_1 \quad x_2]A[x_1 \quad x_2]^T$ (5%)

(b). Using (a), find the points on the ellipse Q=128 that have the farthest distance from the origin. (10%)

(下面還有題目)

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Problem 4 (20%)

Use the Fourier series method to solve the problem:

$$u_t = u_{xx} \qquad 0 < x < 2\pi, t > 0$$
$$u_x(0,t) = u_x(2\pi,t) = 0, \qquad t > 0$$
$$u(x,0) = 2 - 3\cos(x) + 6\cos(4x), \qquad 0 < x < 2\pi$$

Problem 5 (20%)

(1). (a). Suppose that $f(z)$ is a complex function.
What is the definition if $f(z)$ is analytic at a point z_0? (5%)

(b). Consider the complex function $f(z) = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$
What is the domain in which $f(z)$ is analytic. (5%)

(2). What is the Cauchy principal value of the integral $I = \int_{-\infty}^{\infty} \frac{\sqrt{2}}{x^4+1} dx$? (10%)