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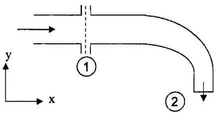
Prob. 1 (20%)

Water flows steadily through the 90° reducing elbow shown in the diagram. At the inlet to the elbow, the static pressure is P_1 and the cross-sectional area is A_1. At the outlet, the cross-sectional area is A_2 and the averaged velocity is V_2. The elbow discharges to the atmosphere with pressure P_{atm}.



(1) Determine the force required to hold the elbow in place. (10%)

(2) Suppose viscous effect is taken into account and boundary layer develops in the elbow duct. How will this viscous effect affect your solution of (1)? State your reason by assuming a velocity profile at the outlet. (10%)

Prob. 2 (20%)

The continuity and Navier-Stokes equations for 2D Newtonian fluid flow are given below.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho g_x-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}\nabla\cdot\vec{V}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\rho g_y-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}\nabla\cdot\vec{V}\right)\right]$$

(1) For incompressible flow, please rewrite the continuity equation. (5%)

(2) Please reduce Navier-Stokes equations above and write down the Navier-Stokes equations under incompressible flow with constant viscosity. (5%)

(3) Please reduce Navier-Stokes equations above and write down the Navier-Stokes equations under incompressible and inviscid flow. (5%)

(4) Consider Navier-Stokes equations under incompressible flow with constant viscosity in problem (2), find the dimensionless group in terms of characteristic length scale L and velocity V_∞. (5%)

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Prob. 3 (20%)

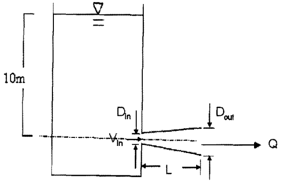
Consider a huge water tank with a special arrangement of exit as shown where the distance from the water level to the center line of exit is 10 m. Assume the exit has a properly rounded inlet (with circular shape and diameter D_{in} = 1 cm), the entrance head loss can be estimated as $H_E = 0.03V_{in}^2/2g$, where V_{in} is the mean inlet velocity and $g = 9.8\,\mathbf{m}/\sec^2$ A straight expansion pipe of length L = 10 cm with inlet diameter D_{in} = 1cm and the exit diameter D_{out} = 3cm is connected to the exit. The head loss of the expansion pipe can be estimated by $H_p = 0.02(L/\overline{D})\,V_{in}^2/2g$, where $\overline{D} = 0.5(D_{in} + D_{out})$

(1) Neglect the head loss by assume frictionless flow, estimate the mean volume flow rate Q through the exit with the expanded connecting pipe. (10%)

(2) Consider the viscous head loss, estimate the mean volume rate Q through the exit with the expansion connecting pipe. (10%)

Hint: Bernoulli equation for steady, incompressible, frictionless flow along a streamline

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = constant$$



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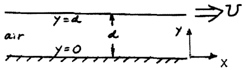
Prob. 4 (20%)

Consider air at room temperature between two infinite plates. See also the drawing below. Assume that the distance between the two plates is d, the bottom plate is fixed, and the top plate is moving at a constant speed U. Find the velocity distribution between the two plates, and the shear stress on the bottom plate if the viscosity of the air is denoted as μ.



Prob. 5 (20%)

Two-dimensional flow with the velocity distribution V(x,y)= (u(x,y), v(x,y)) on a plane shown below. Write down the expression of vorticity of the flow in the direction normal to the plane, z. Also, show the circulation around a very small rectangular contour defined by the points (x_1, y_1), $(x_1+\triangle x, y_1)$, $(x_1, y_1+\triangle y)$, $(x_1+\triangle x, y_1+\triangle y)$. See also the contour in the plot below. Describe the relation between the circulation and the vorticity based on the results obtained.

Note:
V(x,y)=(u(x,y),v(x,y))
u: velocity in x direction
v: velocity in y direction