國立成功大學104學年度碩士班招生考試試題

系所組別：系統及船舶機電工程學系甲乙丙丁組

考試科目：工程數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the general solution of $[x\frac{d}{dx}-1]^3 y=0$. **(10%)**

2. Solve the following equation $y''+2y'+2y=f(t)$ where $f(t)$ is an arbitrary function. **(15%)**

3. Find the Fourier series of $x(t)=t, \quad -\pi<t<\pi, \quad x(t)=x(t+2\pi)$ and calculate the series sum of $1+\frac{1}{4}+\frac{1}{9}+\cdots$ **(15%)**

4. $\vec{v}=a(x+y)\vec{i}+a(y-x)\vec{j}+z^2\vec{k}$,

calculate $\oiint_S \vec{v}\cdot\vec{n}dA$, where $S: x^2+y^2+z^2=a^2$. **(5%)**

5. $\vec{v}=(2xy^2z+\cos y)\vec{i}+(2x^2yz-x\sin y+\sin z)\vec{j}+(x^2y^2+y\cos z)$,

calculate $\oint_C \vec{v}\cdot d\vec{r}_p$ where $C: x=\cos\theta, \quad y=\sin\theta, \quad z=2$. **(5%)**

6. Laplace transform and Fourier transform are defined as

$$X(s)=L[x(t)]\equiv\int_0^\infty x(t)e^{-st}dt, \qquad x(t)=L^{-1}[X(s)]\equiv\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}X(s)e^{st}ds$$

$$X(\omega)=F[x(t)]\equiv\int_{-\infty}^{\infty}x(t)e^{-i\omega t}dt, \qquad x(t)=F^{-1}[X(\omega)]\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}X(\omega)e^{i\omega t}d\omega$$

For a given function $x(t)$, use one simple equation to relate its Laplace transform $L[x(t)]$ and the Fourier transform of an auxiliary function of $x(t)$. Calculate $L[1]$, $L[\delta(t)]$, $F[1]$, $F[\delta(t)]$. **(15%)**

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7. Find the eigenvalues and eigenvectors of the matrix: $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ **(10%)**

8. Use the method of separating variables to find the general solution of Helmhotz equation in polar coordinates $\frac{1}{r}\frac{\partial}{\partial r}\{r\frac{\partial \psi}{\partial r}\}+\frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}+k^2\psi=0$ where k^2 is a constant. **(15%)**

9. Evaluate the following integral $\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dx$ **(10%)**