編號： 135

國立成功大學 105 學年度碩士班招生考試試題

系　　所：系統及船舶機電工程學系

考試科目：工程數學

考試日期：0227，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the response of the damped mass-spring system under a Unit Impulse at time $t=1$.　$y''+3y'+2y=\delta(t-1),\quad y(0)=0,\quad y'(0)=0.$ **(10%)**

2. Find the Fourier series of $f(x)$ as given over one period. **(15%)**

$$f(x)=x^2 \quad \left(-\frac{\pi}{2}<x<\frac{\pi}{2}\right)$$

3. The tank contains 1000 gal(加侖) of water in which initially 100 lb(磅) of salt is dissolved(溶解). Brine(鹵水) runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissolved salt. The mixture in the tank is kept uniform(均勻) by stirring(攪拌). Brine runs out at 10 gal/min. Find the amount of salt in the tank at any time t. **(10%)**

4. Solve the following initial value problem. **(15%)**

$$(x^3D^3-x^2D^2-7xD+16)y=9x\ln(x)$$
$$y(1)=2.5,\quad Dy(1)=4,\quad D^2y(1)=23$$

5. Solve $\frac{\partial^2 w}{\partial x^2}=100\frac{\partial^2 w}{\partial t^2}+100\frac{\partial w}{\partial t}+25w,\ w(x,0)=0$ if $x\geq 0,\ w_t(x,0)=0$ if $t\geq 0,\ w(0,t)=\sin t$ if $t\geq 0,$ by Laplace transforms. **(15%)**

6. Find a general solution in terms of J_ν and Y_ν **(10%)**

$$y''+k^2x^2y=0 \quad \left(y=u\sqrt{x},\ \frac{1}{2}kx^2=z\right)$$

7. Using Green's theorem, evaluate $\int_C \mathrm{F}(r)\bullet dr$ counterclockwise around the boundary curve C of the region R, where $\mathrm{F}=[2x-8y,x+7y],\quad R:\ 16x^2+25y^2\leq 400,\quad y\geq 0.$ **(10%)**

8. Find a general solution.

$$y_1'=-3y_1-4y_2+11t+15$$
$$y_2'=5y_1+6y_2+3e^{-t}-15t-47$$

Make sure your answer for y_2 corresponds to the answer given for y_1.

$y_1(t)=c_1e^t+c_2e^{2t}-2e^{-t}-3t+50$ **(15%)**