

※ 考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1.

(i) Which concept can be used to linearize nonlinear equations or nonlinear systems, and how to realize it? (5%)

(ii) Linearize the following nonlinear system as a linear form  $\Delta\dot{X} = A\Delta X + B\Delta U$  based on the concept in (i) and show us the details of  $A$  and  $B$ . (15%)

$$\dot{X} = F(X, U)$$

where  $X \in R^{n \times 1}$  is the state vector,  $U \in R^{p \times 1}$  is the input vector, and  $F(X, U) \in R^{n \times 1}$  is a function.

2.

(i) Describe the transfer function of the bode plot in Fig. 1. (14%)

(ii) Describe Gain Margin, Phase Margin, and Bandwidth. (6%)

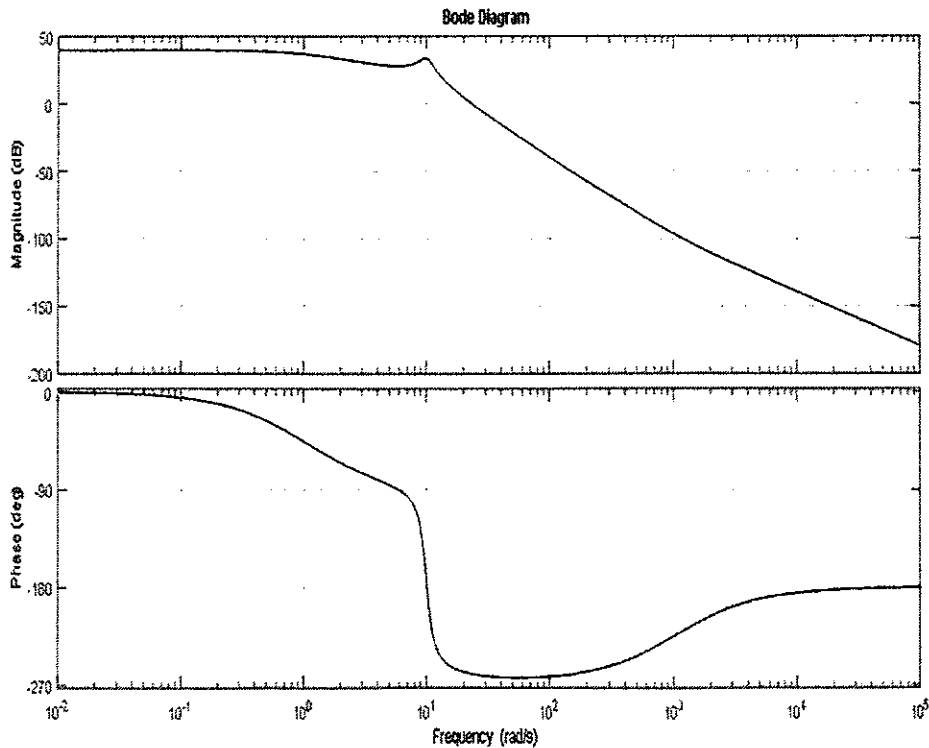


Fig. 1 Bode plot

3.  
 (i) A proportional integral derivative controller is expressed as below:

$$u(t) = K_p e(t) + K_I \int_0^T e(t) dt + K_D \frac{de(t)}{dt}$$

where  $K_p = 0.5$ ,  $K_I = \frac{1}{462}$ , and  $K_D = \frac{1}{20.1}$ .

Please illustrate the circuit of this controller with operational amplifiers (OP amp) and components as resistances, capacitors, and so on. (10%)

- (ii) Find the transfer function  $T(s) = \frac{E(s)}{U(s)}$ . Suppose  $K_p$  is a variable and calculate the sensitivity of  $T(s)$  with respect to  $K_p$ . (10%)

4.  
 (i) Formulate the detailed model of Fig. 2, and calculate the transfer function  $T(s) = \frac{\Theta_m(s)}{E_a(s)}$

between applied voltage  $e_a$  and output motor displacement  $\theta_m$ . (10%)

- (ii) Derive the state space formulation for this system. (10%)

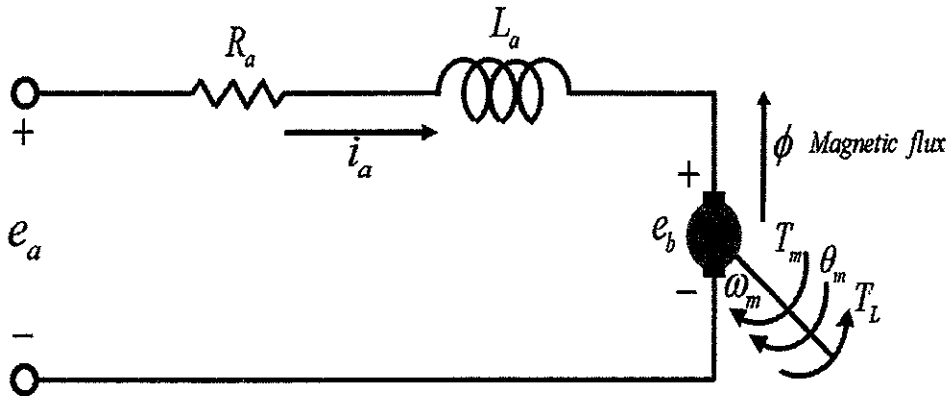


Fig.2 Model of a separately excited DC motor

where  $e_a(t)$ =applied voltage,  $i_a(t)$ =armature current,  $R_a$ =armature resistance,  $L_a$ =armature inductance,  $T_L(t)$ =load torque,  $\phi$ =magnetic flux in the air gap,  $T_m(t)$  =motor torque,  $\omega_m(t)$

=motor angular velocity,  $\theta_m(t)$ =motor displacement,  $J_m$ =motor inertia,  $K_t$ =torque constant,  $B_m$ =viscous-friction coefficient.

5. A linear system with the state space formulation as follows:

$$\dot{X} = AX + BU$$

$$y = CX$$

(i) Derive the steady state error  $e(\infty) = 1 + CA^{-1}B$ , when input  $U$  is a unit step. (10%)

(ii) Derive the steady state error  $e(\infty) = \lim_{t \rightarrow \infty} \left[ (1 + CA^{-1}B)t + C(A^{-1}A^{-1})B \right]$ , when input  $U = t$  (10%)