

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find  $y(x) = ?$  (10%)

$$x^2 y'' + 0.6xy' + 16.04y = 0$$

2. Find the inverse of the transform (10%)

$$\mathcal{L}(f) = \frac{4s - 33}{s^2 + 6s + 34}$$

3. Solve the following ODE. (15%)

$$y' = 2y - 5y^2$$

4. Solve the equation by Laplace transform, where  $y(0) = 1$ ,  $y'(0) = 1$ . (15%)

$$y'' - y = 1$$

5. Find the Laplace integral from the Fourier sine integral of  $f(x) = e^{-kx}$ , where  $x > 0$  and  $k > 0$ . (25%)

6. Heat flow in a plate. The faces of the thin square plate in Fig. 1 with side  $a = b = 6$  are perfectly insulated. The upper side is kept at  $25^\circ\text{C}$  and other sides are kept at  $0^\circ\text{C}$ . Find the steady-state temperature  $u(x, y)$  in the plate.

(Hint:  $u(x, y) = \sum_{n=1}^{\infty} A_n^* \sin\left(\frac{(2n-1)\pi}{a} x\right) \sinh\left(\frac{(2n-1)\pi}{b} y\right)$ ,

$$A_n^* = \frac{2}{a \sinh[(2n-1)\pi b/a]} \int_0^a f(x) \sin\left(\frac{(2n-1)\pi}{a} x\right) dx$$

(25%)

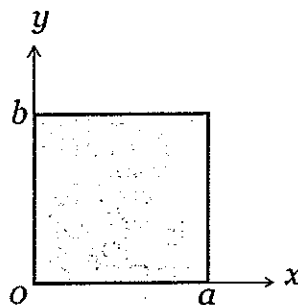


Fig. 1