

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A linear system not described in phase-variable form is expressed as the following:

$$\begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$$

Assume that this system has the controllability matrix  $C_{Mz}$  and can be transformed into the phase-variable ( $x$ ) representation with a controllability matrix  $C_{Mx}$  by using the following transformation

$$z = Px$$

(i) Derive the transformation matrix  $P$  based on the controllability matrices  $C_{Mz}$  and  $C_{Mx}$ .

(15%)

(ii) Find the transformation  $P$  for the following transfer function

(10%)

$$G(s) = \frac{(s+4)}{(s+1)(s+2)(s+5)}$$

2. Design an integral controller to let the following system have a zero steady-state error, 10% overshoot and a settling time of 0.5 second with respect to a unit step input. (25%)

$$\begin{cases} \dot{z} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y = [1 \quad 0]z \end{cases}$$

3. A mathematical model for HIV infection is described as

$$\frac{dT}{dt} = s - nT - (1 - u_1)\beta T v$$

$$\frac{dT^*}{dt} = (1 - u_1)\beta T v - \mu T^*$$

$$\frac{dv}{dt} = (1 - u_2)\kappa T^* - cv$$

The interested number of free HIV viruses  $v$  is selected as the system output. Besides,  $n, \kappa, s, \beta, c,$  and  $\mu$  are constant system parameters, and  $u_1$  and  $u_2$  are control inputs.

(i) Derive the linearized state-space form of this nonlinear HIV model with the operation point

$$(T_0, T_0^*, v_0) = \left( \frac{c\mu}{\beta\kappa}, \frac{s}{\mu} - \frac{cd}{\beta\kappa}, \frac{s\kappa}{c\mu} - \frac{d}{\beta} \right) \text{ and } (u_{10} = u_{20} = 0) \text{ as}$$

$$\begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$$

and express  $A$ ,  $B$ , and  $C$ . (10%)

(ii) Find the closed-loop transfer function (10%)

$$T(s) = \frac{Y(s)}{U(s)}$$

(iii) Replace  $n=0.021/\text{day}$ ,  $\kappa=100$  counts/cell,  $s=10/\text{mm}^3/\text{day}$ ,  $\beta=2.4 \times 10^{-5}/\text{mm}^3/\text{day}$ ,  $c=2.4/\text{day}$ , and  $\mu=0.24/\text{day}$  in  $T(s)$ , and show the exact form of  $T(s)$  with these system parameters. (5%)

4. Implement the PID controller expressed as the following transfer function via using only one operational amplifier, some capacitors, and some resistors. (25%)

$$G_{PID}(s) = \frac{(s+55.92)(s+0.5)}{s}$$