※ 考生請注意：本試題可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please calculate the range of K which stabilizes the system as shown in Figure 1. (15%)

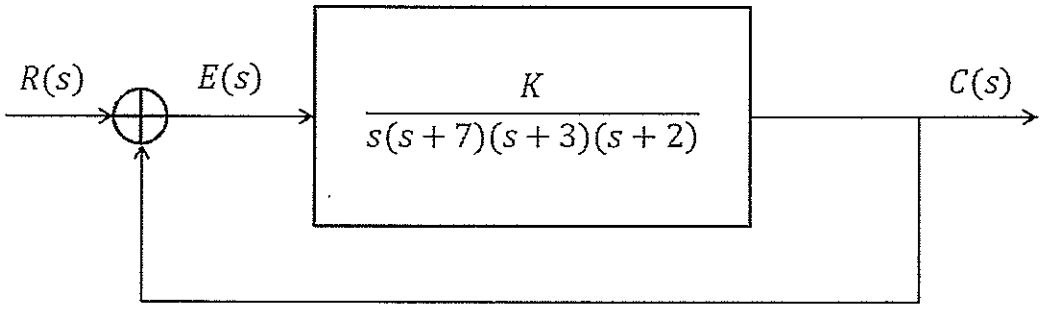

Figure 1 A closed-loop system with an unknown parameter K

2. Consider the circuit of a controller illustrated in Figure 2 and answer the following questions:

(a) Find the transfer function $V_o(s)/V_i(s)$ of this controller. (10%)

(b)Calculate the controller $V_o(s)$, where $R_1 = 100\text{K}\Omega$, $R_2 = 100\text{K}\Omega$, $C_1 = 60\mu F$ $C_2 = 60\mu F$, $V_i(t) = 10\text{V}$. (5%)

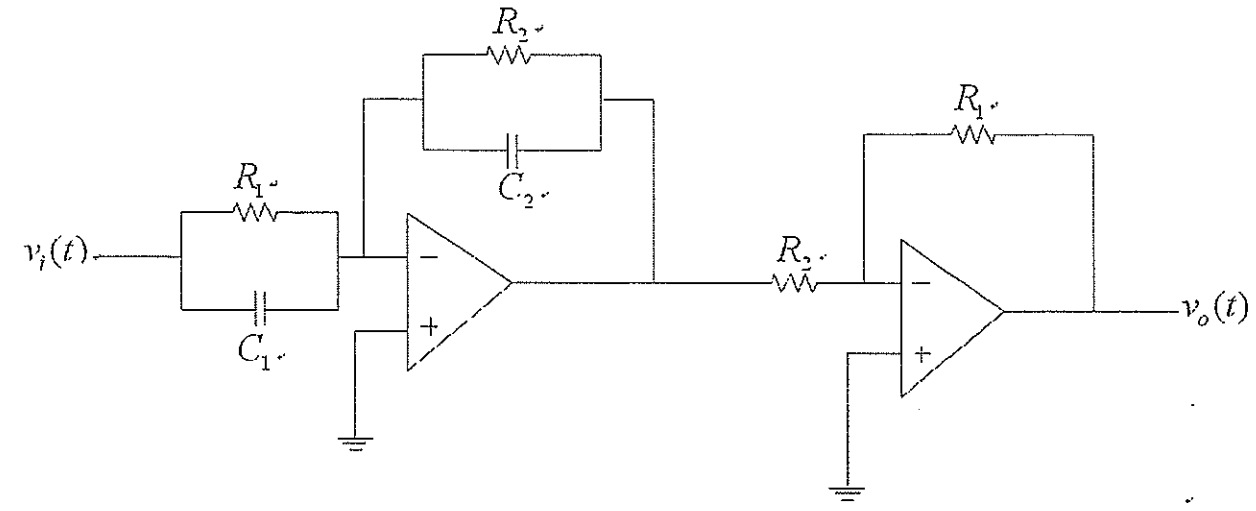

Figure 2 Circuit of a controller

3. Please derive the state space formulation for the DC motor model as that illustrated in Figure 3. (15%)

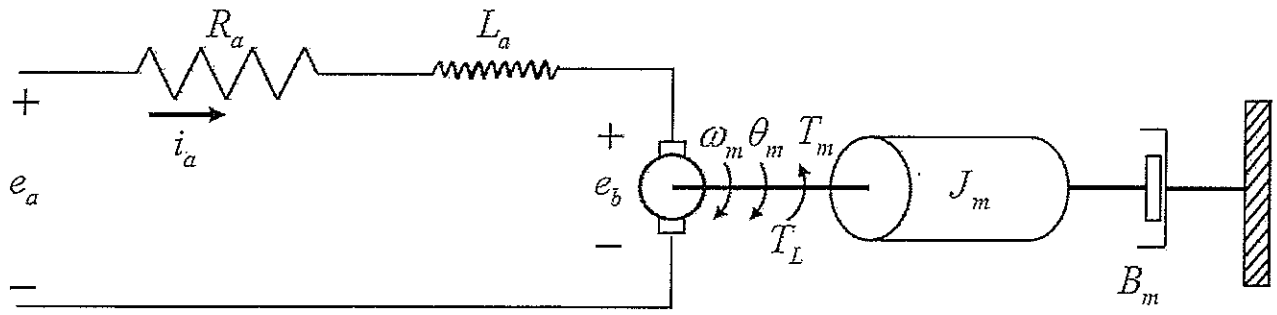

Figure 3 Illustration of a separately excited motor

4. The dynamic equation of a system is given as below.

(a) Determine whether the system is controllable. (5%)

(b)Design a state feedback law $u=-kX$ which generates a closed-loop system with eigenvalues -1, -3 -4. (10%)

$$\dot{X}=\begin{bmatrix}-1 & 0 & 3\\ 2 & -1 & -1\\ -3 & 1 & -2\end{bmatrix}X+\begin{bmatrix}1\\0\\0\end{bmatrix}u,\quad y=\begin{bmatrix}1 & 2 & 1\end{bmatrix}X$$

5. Consider the closed-loop system represented in the state space as follows:

$$\dot{X}=AX+Br$$
$$y=CX$$

(a) Define the steady state solution, X_{ss} for X is,

$$X_{ss}=\begin{bmatrix}V_1 & V_2 & \cdots & V_n\end{bmatrix}^T$$

Giving a unit step function $r=1$ as the input, please derive the steady state error: $e_{ss}=1+CA^{-1}B$. (10%)

(b)Define the steady state solution, X_{ss} for X is,

$$X_{ss}=\begin{bmatrix}V_1t+W_1 & V_2t+W_2 & \cdots & V_nt+W_n\end{bmatrix}^T$$

Giving a ramp step function $r=t$ as the input, please derive the steady state error:

$$e_{ss} = \lim_{t\to\infty}\left[\left(1+CA^{-1}B\right)t + CA^{-1}A^{-1}B\right]. \text{ (10\%)}$$

6. Consider a plant which is controlled by a state feedback design as below:

$$\dot{X} = AX + Bu$$

$$y = CX$$

where

$$A=\begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix},\ B=\begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C=\begin{bmatrix} 1 & 0 \end{bmatrix}, \text{ and } u=-KX$$

(a) Please derive the state space formulation for the system which is controlled by the PI type control design as shown in Figure 4. (10%)

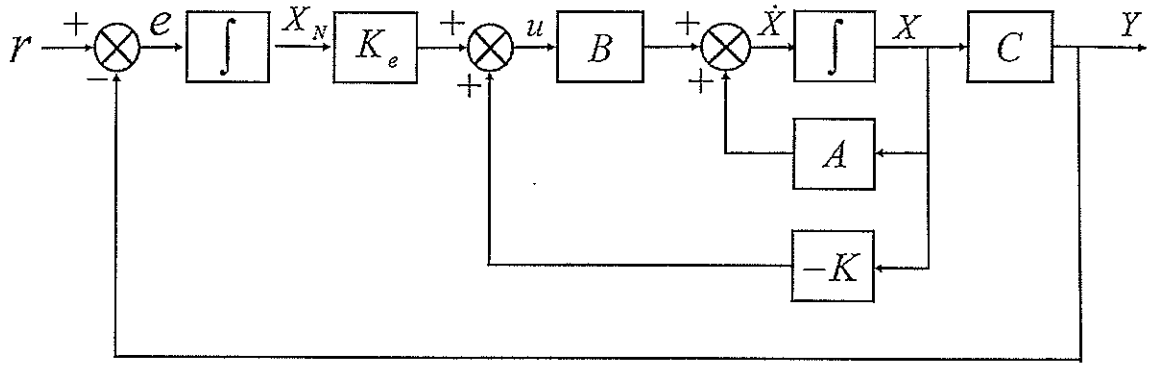

Figure 4 PI type Control Design for a closed-loop system

(b) Based on the system parameters (A,B,C) as depicted above, please find out K and K_e with roots of the desired characteristic equation are $s=-1$, $s=-2$, and $s=-3$. (10%)