

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider a linear system as below

$$\begin{aligned}\dot{X} &= AX + Bu + Br \\ y &= CX\end{aligned}$$

Please answer the following questions:

- (i) How to verify the controllability and observability of this linear system? (10%)
 (ii) Prove the Separation principle of estimation and control. (15%)

2. A state-space form is described as follows:

- (i) Calculate the steady state error of this system which has a unit step input. (10%)

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r \\ y &= [1 \ 0 \ 0]x\end{aligned}$$

- (ii) Stabilize this system via using a state feedback controller $u = -Kx$ and determine the value of K . (15%)

3. (i) Giving a set of initial conditions: $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 1$ and $r(0) = 2$, please linearize the following nonlinear system around the operation point: $(x_1(0), x_2(0), x_3(0))$ and $r(0)$. (13%)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_1^2 x_2 x_3 \\ x_1 x_2 r \\ x_3 r^2 \end{bmatrix}$$

$$y = x_1$$

- (ii) Calculate the transfer function: $G(s) = \frac{Y(s)}{R(s)}$. (12%)

4. (i) Derive the state space form for the following system (10%)

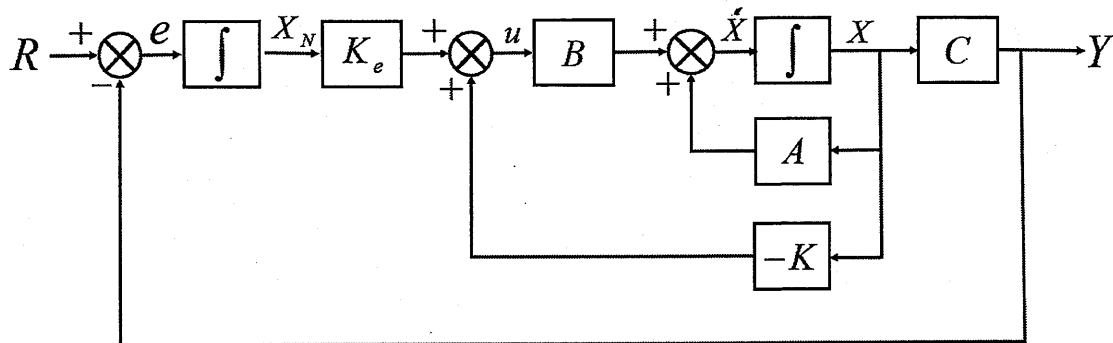


Figure 1 A closed-loop control system

(ii) What is the type of the controller applied in this control system? Why? (5%)

(iii) Giving system parameters (A, B, C) as below, please find out K and K_e with roots of the desired characteristic equation are $s = -1$, $s = -3$, and $s = -5$. (10%)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad C = [1 \quad 0]$$