國立成功大學

114學年度碩士班招生考試試題

編　號：103

系　所：系統及船舶機電工程學系

科　目：自動控制

日　期：0210

節　次：第 2 節

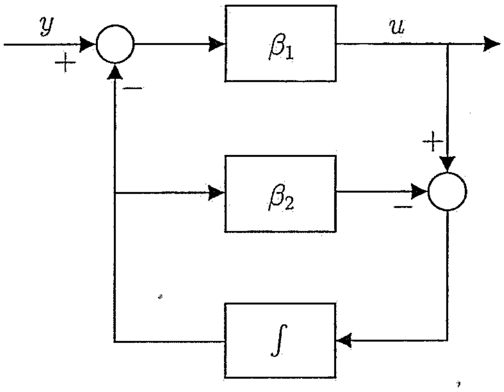
注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (25%) Consider an uncertain linear system $\dot{x} = Ax$ with

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -qk_1 & -qk_2 & -qk_3 \end{pmatrix}$$

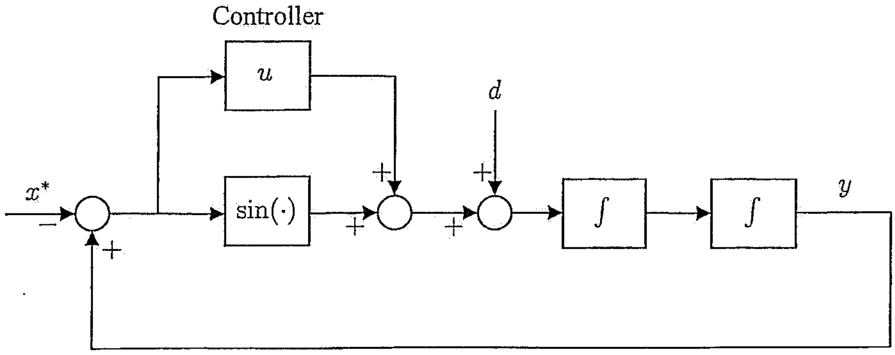
where $k_1 \in \mathbb{R}$, $k_2 \in \mathbb{R}$, and $k_3 \in \mathbb{R}$ are known positive constants while $q \in \mathbb{R}$ is an unknown paramater. Suppose that q satisfies $1 \leq q \leq \overline{q}$ where $\overline{q} \in \mathbb{R}$ is a known positive constant. Please explain, along with a rigorous proof, the conditions for k_1, k_2, and k_3 so as to ensure that A is Hurwitz.

2. (25%) A controller with the structure shown below is frequently used in linear control systems. Suppose that $y \in \mathbb{R}$ represents the measurement from the system output, and $\beta_1 \in \mathbb{R}$ and $\beta_2 \in \mathbb{R}$ are positive constants.



Provide a rigorous proof to analyze and determine the working function of this controller in the frequency domain.

3. (25%) Consider a feedback control system as described below, where $x^* \in \mathbb{R}$ represents the regulation command and $d \in \mathbb{R}$ denotes the unknown external disturbance, both of which are assumed to be fixed constants.

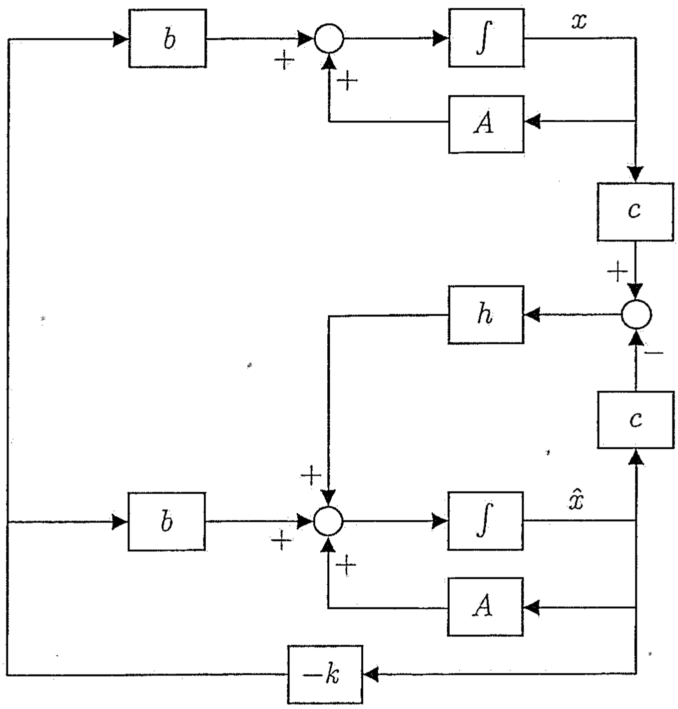


Suppose that $y \in \mathbb{R}$ is the measurement output of the system, and the controller $u(\cdot)$ is constructed as

$$u(t) = -g_1 \left(y(t) - x^*\right) - g_2 \frac{dy(t)}{dt} - g_3 \int_0^t \left(y(s) - x^*\right)\, ds$$

where $g_1 \in \mathbb{R}$, $g_2 \in \mathbb{R}$, and $g_3 \in \mathbb{R}$ are positive constants. Derive and determine the values of g_1, g_2, and g_3 such that the closed-loop system ensures the performance $y(t) \to x^*$ as $t \to \infty$.

4. (25%) Consider a linear system consisting of the following structure, where $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $h \in \mathbb{R}^n$, $k^T \in \mathbb{R}^n$, and $c^T \in \mathbb{R}^n$ are all constant matrices.



Letting $\mathcal{Z} = (x^T, \hat{x}^T) \in \mathbb{R}^{2n}$ with $x \in \mathbb{R}^n$ and $\hat{x} \in \mathbb{R}^n$ readily implies that the system can be described in a compact form $\dot{\mathcal{Z}} = \mathcal{F}\mathcal{Z}$.

(i) (10%) Determine $\mathcal{F} \in \mathbb{R}^{2n\times 2n}$ in terms of A, b, h, k, and c.

(ii) (15%) Provide a rigorous proof to show that $\mathcal{F}$ is Hurwitz if and only if $A - bk$ and $A - hc$ are Hurwitz.