

成功大學造船工程所 七十九學年度碩士班 工程數學 考題

(5) 1. Solve  $u_{xx} + u_{yy} = 0$ ,  $0 < x < 1$ ,  $0 < y < 1$

$u(0, y) = u(1, y) = 0$

$u(x, 0) = \sin \pi x$

$u(x, 1) = \sin 2\pi x$

(3) 2. Integrate  $\oint_C \frac{dz}{z^2 - 1}$ , where  $C$  is the circle  $|z| = 2$

(7) 3. Find Fourier series of  $f(x) = |x|$ ,  $-\pi \leq x \leq \pi$ , then solve following two series

(a)  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = ?$

(b)  $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \dots = ?$

(5) 4. A circular helix is given by  $\vec{r}(t) = a \cos t \vec{i} + a \sin t \vec{j} + ct \vec{k}$ . derive the following quantities along this curve :

- a) velocity and acceleration vectors;
- b) unit tangent, normal, and binormal vectors;
- c) radius of curvature and radius of torsion.

(10) 5. If  $\vec{r}$  and  $\vec{n}$  are the position vector and unit normal vector to a closed surface  $S$  which can be non-smooth, evaluate the surface integral of

$$\oint_S \frac{\vec{r}}{r^3} \cdot \vec{n} \, ds,$$

- when the origin of coordinates is
- a) outside of  $S$ ,
  - b) inside of  $S$ ,
  - c) on  $S$ .

(5) 6. Derive the orthogonal trajectory of  $x^2 + (y-c)^2 = c^2$ . Can this orthogonal trajectory be derived from the Cauchy-Riemann Equation? Explain.

(5) 7. Solve the first order simultaneous ordinary differential equations:

$$\vec{X}' = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix} \vec{X}$$

(5) 8. Solve  $x^3 y'' + xy' - y = 0$

(5) 9. Solve  $x^2 y'' + x^2 y' - 2y = 0$

(10) 10. Solve  $y'' + \lambda y = 0$ ,  $-d \leq x \leq 0$ ,

$y'(-d) = 0$ ,

$y'(0) - v y(0) = 0$ ,

where  $v$  is a positive constant. Discuss the orthogonal property of this boundary value problem.

(5) 11. Evaluate the Laplace transform of

$$\int_0^t \frac{\sin u}{u} du$$

(5) 12. Evaluate the integral

$$\int_0^{\infty} \frac{x^{1/2}}{1+x^2} dx$$

(10) 13. If  $f(z)$  is analytic inside and on a closed contour  $C$  which can be non-smooth, evaluate the line integral of

$$\oint_C \frac{f(z)}{z-a} dz,$$

when  $a$  is

- a) outside of  $C$
- b) inside of  $C$
- c) on  $C$ .

(5) 14. Derive a governing differential equation which describes the small vibration of a heavy hanging chain of length  $L$ .

(15) 15. Solve  $U_{tt} - c^2 U_{xx} = 0$ ,  $0 \leq x \leq L$ ,  $0 \leq t$

$$U(x,0) = \sin \pi x/L$$

$$U_t(x,0) = 0$$

$$U(0,t) = 0$$

$$U(L,t) = \sin t$$