#1. (15 分)

Find the solution of the problem

$$u_{tt} - u_{xx} + u = 0 \qquad (0 < x < \pi,\ t > 0)$$
$$u(x,0) = f(x) \qquad (0 \le x \le \pi)$$
$$u_t(x,0) = 0 \qquad (0 \le x \le \pi)$$
$$u(0,t) = 0 \qquad (t \ge 0)$$
$$u(\pi,t) = 0 \qquad (t \ge 0)$$

where $f(x) = \begin{cases} x & (0 \le x \le \frac{\pi}{2}) \\ \pi - x & (\frac{\pi}{2} \le x \le \pi) \end{cases}$

#2 (10分)

Evaluate $\int_0^{2\pi} (a + b\sin\theta)^{-1} d\theta$ where $|a| > |b|$

#3 (5分)

Evaluate $\oint_C \frac{\eta\, d\eta}{\eta^2 - z^2}$, whenever it exists, if C is the unit circle in the positive sense.

#4 (5分)

Find λ such that the set

$$\begin{cases} 2x_1 - 2x_2 + x_3 = \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 = \lambda x_2 \\ -x_1 + 2x_2 = \lambda x_3 \end{cases}$$

possesses a nontrivial solution. Also obtain the general solution in each case.

#5 (10分)

$$\frac{d^2y}{dx^2} + \pi^2 y = a + x \quad , \quad y(0) = 0, \quad y(1) = 0$$

Determine whether there is any value of the constant, a, for which the problem has a solution, and find the solution.

#6 (5分)

Find the Fourier series of $f(x) = \cos cx$, $-\pi \le x \le \pi$ where c may or may not be an integer.

#7 (8分)

Solve the particular solution of $y''' - 7y' + 6y = f(x)$

#8 (15分)

Solve $x^2y'' + 3xy' + (1+x)y = 0$

#9 (7分)

Solve $\begin{cases} x' + y'' = 0 \\ x' + x + y' = 0 \end{cases}$

#10 (10分)

Find $\mathcal{L}\left(\frac{\cos\sqrt{t}}{\sqrt{t}}\right)$

#11 (10分)

Calculate the line integral of $2y\vec{i} + 3\vec{j} + 3y\vec{k}$ along the intersection curve of the following two surfaces $\begin{cases} x^2 + y^2 + z^2 = 6z \\ z = x + 3 \end{cases}$