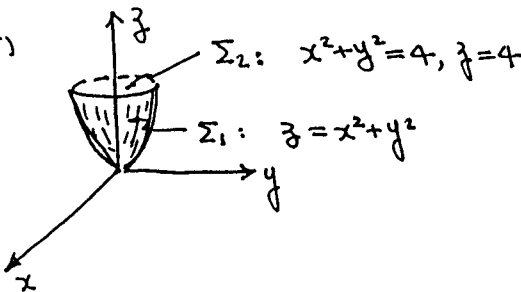1. (10分) Solve $y' + (6 + \frac{2}{x})y + \frac{8}{x} = 0$

2. (10分) Solve $\vec{x}' = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix} \vec{x}$

3. (5分) Given $\int \frac{dx}{x(1+x^2)} = \ln \frac{x}{\sqrt{1+x^2}}$, Find the Laplace Transforms of

$\frac{1-\cos t}{t}$ and $\int_0^t \frac{1-\cos u}{u} du$

4. (5分)

$\Sigma_2: x^2 + y^2 = 4, z = 4$

$\Sigma_1: z = x^2 + y^2$

Given $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$

Calculate $\iint_{\Sigma_1} \vec{F} \cdot \vec{n} \, d\sigma$

5. (10分) Find the Frobenius Series Solution of $x^2 y''' + y = 0$

6. (10分) A rectangular plate of length a, width b, is subjected to a uniform lateral load P_0. Given that the lateral displacement $w(x,y)$ is governed by $\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P_0}{D}$ where D is a material constant, Solve $w(x,y)$ by Double Fourier Sine Series Method.

7. (20分) Evaluate the integral $\int_0^\infty \frac{x^{m-1}}{x+1} dx$ $(0 < m < 1)$

Given $\begin{cases} u_{tt} - c^2 u_{xx} = 0 \\ u(x,0) = \sin x \qquad u(0,t) = 0 \\ u_t(x,0) = \sin 2x \qquad u(\pi,t) = 0 \end{cases}$

8. (10分) Solve the Problem by Method of characteristics

9. (10分) Solve the Problem by Method of seperation of Variables

10. (10分) Solve the Problem by Method of Laplace Transform