

1. (10分) Solve $(1-x^2)y'' - 2xy' = 0$

2. (10分) Solve $x^2y'' - 2xy' + 2y = \frac{24}{x^2}$

3. (15分) Let $\delta(t) =$ unit impulse function

$u(t) =$ unit step function

$F(s) =$ Laplace transform of $f(t) = \mathcal{L}[f(t)]$

(a) Find $\mathcal{L}[\delta(t-a)]$

$\mathcal{L}[e^{at}f(t)]$

$\mathcal{L}[f(t-a)u(t-a)]$

(b) Solve
$$\begin{cases} y'' + 2y' + 2y = \delta(t-1) \\ y(0) = 1 \\ y'(0) = -1 \end{cases}$$

4. (5分) $\vec{a} = \vec{i} - \vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} - \vec{k}$, $\vec{c} = -\vec{i} - \vec{j} + \vec{k}$

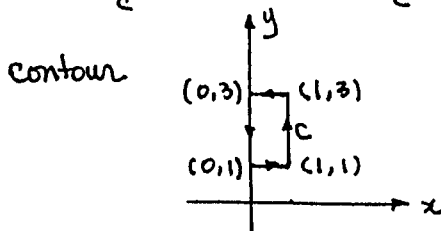
$\vec{v} = 3\vec{i} + 2\vec{j} + \vec{k}$,

Express \vec{v} as a linear combination of $\vec{a}, \vec{b}, \vec{c}$

5. (10分) $\vec{F} = (y - e^x)\vec{i} + (\cos 2y - x)\vec{j}$

$\vec{G} = (y - e^x)\vec{i} + (\cos 2y + x)\vec{j}$

Calculate $\oint_c \vec{F} \cdot d\vec{r}$ and $\oint_c \vec{G} \cdot d\vec{r}$ where c is the closed



6. (10分) Integrate $\frac{15z+9}{z^3-9z}$ counterclockwise around the following paths

(a) $|z| = 1$

(b) $|z| = 4$

7. (20分) Calculate $\int_0^{\infty} \frac{\sin x}{x} dx$

8. (20分) Solve
$$\begin{cases} u_t - k u_{xx} = 0 & , 0 < x < l, t > 0 \\ u(0, t) = 0 & , t \geq 0 \\ u(l, t) = 0 & , t \geq 0 \\ u(x, 0) = x(l-x) & , 0 \leq x \leq l \end{cases}$$