

- (10%) 1. Why does the efficiency of a propeller eventually begin to drop as the number of blades is increased?
- (10%) 2. Does the viscosity of (a) liquids, and (b) gases, increase or decrease with temperature? Why?
- (5%) 3. Explain the physical meanings of (a) Reynolds number, (b) Froude number, and (c) Mach number.
- (10%) 4. (a) Write down the Navier-Stokes equation in vector form.  
(b) Explain the physical meaning of each term in (a).
- (10%) 5. In model studies of gravitational effects, velocities are reduced in proportion to the square root of the scale ratio. (a) What velocity change is required in model studies of viscous effects? (b) What practical difficulty would such a relationship introduce in model studies involving both gravitational and viscous effects?
- (5%) 6. When the valve on a drinking fountain is opened, the jet at once rises higher than its final level. Why does this not agree with the general equation for flow establishment?
- (6%) 7.(a) An incompressible velocity field is given by  

$$u = a(x^2 - y^2) \quad \text{where } v \text{ unknown, } w = b$$
 where a and b are constants. What must the form of the velocity component v be?
- (6%) 7.(b) Take the velocity field of 7.(a) with  $b=0$  for algebraic convenience  

$$u = a(x^2 - y^2) \quad v = -2axy \quad w = 0$$
 and determine under what conditions it is a solution to the Navier-Stokes momentum equation. Assuming that these conditions are met, determine the resulting pressure distribution when z is "up" ( $g_x = 0, g_y = 0, g_z = -g$ ).
- (6%) 7.(c) Does a stream function exist for the velocity field of 7.(a)  

$$u = a(x^2 - y^2) \quad v = -2axy \quad w = 0$$
 If so, find it and plot it and interpret it.
- (6%) 7.(d) Does a velocity potential exist for the velocity field of 7.(a)  

$$u = a(x^2 - y^2) \quad v = -2axy \quad w = 0$$
 If so, find it and plot it and compare with part 7.(c).
- (12%) 8. Air at standard conditions is flowing between the parallel flat plates as shown. The velocity is uniform at the entrance and has a magnitude,  $U_0 = 25 \text{ m/sec}$ . A boundary-layer trip on each wall at the channel inlet assures that a turbulent boundary layer develops from the leading edge. The boundary-layer velocity profile and thickness may be approximated by the expressions.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \quad \frac{\delta}{x} = 0.370 \left(\frac{\nu}{U_0 x}\right)^{1/4}$$

where U is the core velocity. Note that U is a function of x. The width of the plates, w, is much larger than the distance between the plates, h, and therefore end effects may be neglected. Determine the pressure drop between the inlet and a point 5 m downstream of the inlet.

9. Consider two-dimensional laminar flow along a flat plate. The velocity profile the boundary layer is assumed to be sinusoidal, that is :

$$\frac{u}{U} = \sin\left(\frac{\pi y}{2\delta}\right)$$

Find expressions for :

- (4%)(a) The rate of growth of  $\delta$  as a function of  $x$ .  
(5%)(b) The displacement thickness,  $\delta^*$ , as a function of  $x$ .  
(5%)(c) The total friction force on a plate of length,  $L$ , and width,  $b$ .