

(1) (13 分) Find the ^{general} solution $y(x)$ of the following equation:

$$x^2 y'' + x y' - y = \frac{1}{x}$$

(2) (13 分) Find the ^{general} solution $y(x)$ of the following equation:

$$y' + \frac{y}{x} = (\ln x) y^2 ; y(1) = 1$$

(3) (12 分) Find the inverse ^{transform} of the following transform:

$$\frac{e^{-s} + e^{-2s}}{s^2 - 3s + 2}$$

(4) (12 分) Find the solution $y(x)$ of the following equation:

$$y'' + y = \sin x ; y(0) = y'(0) = 0$$

(10分)(5). Find $\oint_C \frac{e^{2iz}}{z^2(z^2+2z+2)} dz$ where C is the circle of $|z|=4$

(6). Find the general solution in terms of Bessel's functions for the following equations

(10分) (a) $x^2 y'' + x y' + (\lambda^2 x^2 - \nu^2) y = 0$

(10分) (b) $\frac{1}{2} x^2 y'' + \frac{1}{2} x y' + (2x^4 - \frac{1}{8}) y = 0$

(20分)(7). Find the steady-state oscillation corresponding to $y'' + cy' + y = r(t)$ where $c > 0$ and

$$r(t) = \begin{cases} t + \frac{\pi}{2} & \text{if } -\pi < t < 0 \\ -t + \frac{\pi}{2} & \text{if } 0 < t < \pi \end{cases} \quad r(t + 2\pi) = r(t)$$