

<10分> (1) Solve $y(t) = t + \int_0^t y(\tau) \sin(t-\tau) d\tau$

<10分> (2) if Fourier Transform of $f(t)$ is defined as

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Find the Fourier Transform of

$$f(t) = \begin{cases} (1 - \frac{|t|}{T}) \frac{1}{T} & , |t| < T \\ 0 & , |t| > T \end{cases}$$

(25分) (3) Find the eigenvalues and eigenfunctions of the following Boundary value Problem

$$\begin{cases} y'' + \lambda y = 0 & , 0 \leq x \leq h \\ y' = 0 & , x = 0 \\ y' - ay = 0 & , x = h \end{cases}$$

where a is a positive constant. Then

Find the eigenfunction expansion of 1.

(5分) (4) Find $\oint_C \vec{F} \cdot d\vec{r}$

where $\vec{F} = (y^2 - 7y)\vec{i} + (2xy + 2x)\vec{j}$

$C = \text{unit circle} : x^2 + y^2 = 1$

(背面仍有題目.請繼續作答)

(20分)(5) Solve the following boundary-value problem

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ u(x, 0) = 1 \\ u(1, y) = 0 \\ u(x, 1) = 0 \\ u(0, y) = 1 \end{cases}$$

(10分)(6) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2-\cos\theta}}$

(10分)(7) Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$

(10分)(8) Evaluate $\int_{-\infty}^{\infty} \frac{\sin \pi x}{x-x^5} dx$