

1) The followings are the equations of the vibration as shown in Figure 1.

(20%)

$$m_1 \ddot{y}_1 = -k_1 y_1 + k_2 (y_2 - y_1)$$

$$m_2 \ddot{y}_2 = -k_2 (y_2 - y_1)$$

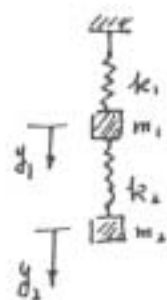


Figure 1.

Please find the eigenvalues and the eigenvectors of the above equations.

(Assume $m_1 = m_2 = 1$, $k_1 = 3$, $k_2 = 2$)

2) Find the Fourier Series of the following function

(10%)

$$f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \end{cases}$$

3) Find the Fourier Integral of the following function

(10%)

$$f(x) = \begin{cases} 1, & -4 < x < 4 \\ 0, & x > 4, x < -4 \end{cases}$$

4) Solve the following differential equations.

(5%) (i) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + [(ax)^2 - 16] y = 0$

(5%) (ii) $4x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + (x - \frac{1}{36}) y = 0$

5). Find the general solution of $y'' + 3y' = 28 \cosh 4x$.

(10%)

6). Solve the following problem by the method of separating variables:

(20%)

$$\frac{\partial^3 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 \leq x \leq a, \quad 0 \leq y \leq b, \quad t \geq 0,$$

$$u(0, y, t) = 0,$$

$$u(a, y, t) = 0,$$

$$u(x, 0, t) = 0,$$

$$u(x, b, t) = 0,$$

$$u(x, y, 0) = 1,$$

$$\left. \frac{\partial u}{\partial t} \right|_{t=0} = 1.$$

7). Find all roots of $\sqrt[3]{216}$.

(5%)

8). Evaluate the following the following integrals:

(15%)

$$(1) \int_0^{2\pi} \frac{1 + \sin \theta}{3 + \cos \theta} d\theta$$

$$(2) \int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 4)^2}$$

$$(3) \int_{-\infty}^{\infty} \frac{dx}{x^2 - ix}$$