

- (05%) 1. Find the lowest-order differential polynomial  $P(D)$  such that

$$P(D)[x^2 \cos kx] = 0, \text{ where } D = \frac{d}{dx}.$$

- (05%) 2. Find the eigenvalues of the matrix: 
$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- (15%) 3. Find the general solution of  $[\frac{d^2}{dx^2} - \frac{2}{x^2}]^2 y = 0$ .

- (10%) 4. If Fourier transform of  $f(t)$  is defined as  $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$ ,

$$\text{Find the Fourier transform of } f(t) = \begin{cases} 1/2T, & |t| < T \\ 0, & |t| > T \end{cases}$$

What are the functions  $\lim_{T \rightarrow 0} f(t)$  and  $\lim_{T \rightarrow 0} F(\omega)$ ?

- (10%) 5. Given  $\omega = \sqrt{k/m}$ ,  $\xi = c/2m\omega$ ,  $\omega_D = \omega\sqrt{1-\xi^2}$ ,  $4mk > c^2$ ,

Solve the following equation by Laplace transform:

$$my'' + cy' + ky = f(t), \quad y(0) = 0, \quad y'(0) = 0.$$

- (05%) 6. If  $\vec{F} = a(x+y)\vec{i} + a(y-x)\vec{j} + z^2\vec{k}$

Calculate  $\oiint_S \vec{F} \cdot \vec{n} dA$ , where  $S: x^2 + y^2 + z^2 = a^2$ .

- (20%) 7. Solve the following problem by the method of separating variables:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x \leq a, \quad 0 \leq y \leq b$$

$$u(x, 0) = 0,$$

$$u(a, y) = 0,$$

$$u(x, b) = 0,$$

$$u(0, y) = 1.$$

- (10%) 8. Find all roots of  $1^{1/4}$  in the complex plane.

- (20%) 9. Evaluate the following integrals:

$$(1) \int_0^{2\pi} \frac{d\theta}{13 - 5 \sin \theta}$$

$$(2) \int_{-\infty}^{\infty} \frac{x}{8 - x^3} dx$$