(15%)

1. Find the general solution of $[x^2 \frac{d^2}{dx^2} - 2]^2 y = 0$.

(15%)

2. Find the eigenvalues and eigenfunctions of the following boundary value problem:
$y'''' + \lambda^2 y'' = 0$, $y(0) = y''(0) = 0$, $y(l) = y''(l) = 0$, where $\lambda^2$ is a constant.

(10%)

3. Given $\omega = \sqrt{k/m}$, solve the following equation by Laplace transform:
$my'' + ky = f(t)$, $y(0) = 0$, $y'(0) = 0$

(15%)

4. Find the Fourier series of

$$f(x) = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}, \quad f(-x) = -f(x), \quad f(x+2\pi) = f(x)$$

Then evaluate the following series: $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots =$

(10%)

5. If $\vec{V} = y^2\vec{i} + 2(xy+z)\vec{j} + 2y\vec{k}$, calculate $\int_{(0,0,0)}^{(1,1,1)} \vec{V} \cdot d\vec{r}$.

(20%)

6. Use the method of separating variables to find the general solution of Helmhotz equation in polar coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\{r\frac{\partial \psi}{\partial r}\} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + k^2\psi = 0$$ where $k^2$ is a constant.

(15%)

7. Evaluate the following integral $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx$