(10%) #1. Find the general solution of $y'' - 2y' + y = e^t$.

(10%) #2. Find the general solution of $t^3y''' + t^2y'' - 2ty' + 2y = t^{-2}$

(15%) #3. Solve the free vibration problem of an under-damped system:

$my'' + cy' + ky = 0, \quad y(0) = y_0, \quad y'(0) = 0, \quad \text{with } 4mk > c^2.$

(a) Express your solution in terms of $\omega = \sqrt{k/m}$, $\xi = c/2m\omega$, $\omega_D = \omega\sqrt{1-\xi^2}$.

(b) Plot the solution.

(c) Indicate how you can evaluate the two system ratios k/m, c/m if you are conducting an experiment with known value of m.

(05%) #4. Solve the simultaneous differential equations

$$\begin{cases} x' + 5x + y' + 4y = e^{-t} \\ x' + 2x + y' + y = 3 \end{cases}$$

(05%) #5. Find the Laplace transform of $\dfrac{e^{-at} - e^{-bt}}{t}$

(15%) #6. If the Fourier transform of $x(t)$ is defined as

$F\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)\exp(-i\omega t)dt$. Find the Fourier transforms of

(a) $x(t-\tau)$, **(b)** $x(\frac{t}{s})$, **(c)** $\frac{1}{\sqrt{s}}x(\frac{t-\tau}{s})$

(10%) #7. If $\vec{V} = (x^2 + y^2x)\vec{i} + (y^2 + x^2y)\vec{j}$, calculate $\int_{(0,1)}^{(1,2)} \vec{V} \cdot d\vec{r}$.

(20%) #8. Solve the following boundary value problem:

$\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0, \quad 0 \le x \le 1, \quad 0 \le y \le 1$

$u(0,y) = 0, \; u(1,y) = f(y), \; \dfrac{\partial u}{\partial y}(x,0) = 0, \; \dfrac{\partial u}{\partial y}(x,1) = 0$

(10%) #9. Evaluate the following complex integrals

(a) $\int_{1-j\infty}^{1+j\infty} \dfrac{ze^{zt}}{z^2+1}dz$, **(b)** $\int_{1-j\infty}^{1+j\infty} \dfrac{e^{zt}}{z^2+1}dz$