#1) (20%) For the characteristic equation of feedback control system given below, determine the range of K so that the system is asymptotically stable (if possible).

$$S^4 + KS^3 + 5S^2 + 10S + 10K = 0$$

#2) (20%) (i) Construct the root-locus diagram for the following control system for which the open loop transfer function, $G(s)H(s)$, is given as

$$G(s)H(s) = \frac{K(S-1)}{S(S+1)(S+2)}, \quad -\infty < K < \infty \qquad (15\%)$$

(ii) Find breakaway points. (5%)

#3) (20%) (i) Obtain the state-transition matrix, $\phi(t)$, for the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t). \qquad (10\%)$$

(ii) If $u(t)$ is the unit-step function occuring at $t=0$, (or $u(t)=1(t)$) and the initial state is zero, ($\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$), obtain the time response of the above system shown in (i). (10%)

#4) (20%) Consider the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and letting $\begin{cases} \underset{\sim}{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ \underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \underset{\sim}{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \end{cases}$

Find the transfer function between the input $U(s)$ and the output $y(s)$.

#5) (20%) Draw the Bode diagram of the following transfer function:

$$G(jw) = \frac{e^{-0.5jw}}{1+jw}$$

where $0.1\ \text{rad/sec} \le w \le 10\ \text{rad/sec}$.