

NOTE : 1. You can write Chinese or English for your solutions.  
2. You should take 3-digit numbers for your decimal points.

- A bottling company uses a filling machine to fill plastic bottles with a popular cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary according to a normal distribution with mean  $\mu = 298$  ml and standard deviation  $\sigma = 3$  ml. (15 points)
  - What is the probability that an individual bottle contains less than 295 ml?
  - What is the probability that the mean content of the bottles in a four-pack is less than 295 ml?
- The Gallup Poll once asked a random sample of 1540 adults, "Do you happen to jog?" Suppose that in fact 15% of all American adults jog. (10 points)
  - Use the normal approximation to find the probability that between 13% and 17% of the sample jog.
  - What sample size would be required to reduced the standard deviation of the sample proportion to one-half the value in this problem?
- The clearance between a pin and the collar around it is important for the proper performance of a disc drive from small computers. The specifications call for the pin to have diameter 0.525 cm and for the collar to have diameter 0.526 cm. The clearance will then be 0.001 cm. In practice, both diameters vary from part to part independently of each other. The diameter  $X$  of the pin has the  $N(0.525, (0.0003)^2)$  distribution and the distribution of the diameter  $Y$  of the collar is  $N(0.526, (0.0004)^2)$ . What is the probability  $P(Y \leq X)$  that the pin will not fit inside the collar? (10 points)
- In a study of cereal leaf beetle damage on oats, researchers measured the number of beetle larvae per stem in small plots of oats after randomly applying one of two treatments: no pesticide or malathion at the rate of 0.25 pound per acre. The data gave the summary statistics below.

Group	Treatment	n	$\bar{x}$	s
1	Malathion	14	1.86	0.62
2	Control	13	3.47	1.05

Is there significant evidence at the 1% level that the mean number of larvae per stem is reduced by malathion? Be sure to state  $H_0$  and  $H_a$ . (20 points)

- As of July 1, 1988, the New York Mets baseball team had played 37 games at home and 40 games away. They won 26 of their home games and 23 of the games played away. (10 points)
  - Find the standard error needed for testing that the probability of winning is the same at home and away.
  - Is it generally believed that it is easier to win at home than away? What conclusion do you draw?

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6. The gas mileage of an automobile first increases and then decreases as speed increases. Suppose that this relationship is very regular, as shown by the following data on speed (miles per hour) and mileage (miles per gallon):

Speed	20	30	40	50	60
Mileage	24	28	30	28	24

Show that the correlation between speed and mileage is  $r=0$ . Explain why the correlation is 0 even though there is a strong association between speed and mileage. (10 points)

7. The mean height of American women in their early twenties is about 65.5 inches and the standard deviation is about 2.5 inches. The mean height of men the same age is about 68.5 inches, with standard deviation about 2.7 inches. If the correlation between the heights of husbands and wives is about  $r=0.5$ , what is the slope of the regression line of the husband's height on the wife's height in young couples? Predict the height of the husband of a woman who is 67 inches tall. (10 points)
8. In a particular region of the United States the number of small businesses that failed per day over a 100-day period during the last recession was as follows:

										Total
x	0	1	2	3	4	5	6	7	8	36
f	3	10	24	31	20	8	2	1	1	100

Where  $x$  : Number of Failures per Day

$f$  : Number of Days on which Specified Number of Failures Occurred.

Determine whether the Poisson distribution is a good fit or not. Use a 0.01 significance level. (15 points)

Standard Normal Distribution:

$$P(0 < z < 1) = 0.3413 \quad P(0 < z < 1.12) = 0.3686 \quad P(0 < z < 1.16) = 0.3770$$

$$P(0 < z < 2) = 0.4772 \quad P(0 < z < 2.20) = 0.4861 \quad P(0 < z < 1.19) = 0.3830$$

t Distribution:

$$t(0.01, 25) = 2.485 \quad t(0.01, 27) = 2.473 \quad t(0.005, 25) = 2.787 \quad t(0.005, 27) = 2.771$$

F Distribution:

$$F(0.01, 13, 12) = 4.12 \quad F(0.005, 13, 12) = 4.30 \quad F(0.01, 12, 13) = 3.98$$

$$F(0.005, 12, 13) = 4.13$$

Chi-square Distribution:

$$\chi^2(0.01, 4) = 13.28 \quad \chi^2(0.01, 5) = 15.09 \quad \chi^2(0.01, 6) = 16.81 \quad \chi^2(0.01, 7) = 18.48$$

$$\chi^2(0.01, 8) = 20.09 \quad \chi^2(0.005, 4) = 14.86 \quad \chi^2(0.005, 5) = 16.75 \quad \chi^2(0.005, 6) = 18.55$$

$$\chi^2(0.005, 7) = 20.28 \quad \chi^2(0.005, 8) = 21.95$$