(20%) 1. Solve the initial value problem

$$(x^2 + 1)y' + y^2 + 1 = 0 \quad , \quad y(0) = 1$$

by Separable Equation Method

(20%) 2. Solve the initial value problem

$$y'' - 4y' + 4y = 0 \quad , \quad y(0) = 3 \quad , \quad y'(0) = 1$$

by the Characteristic Equation Method

(20%) 3. Solve $y' - y = 0$ by the Power Series Method

$$\left(y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots\cdots = \sum_{m=0}^{\infty} c_m x^m \right)$$

(20%) 4. Using the Laplace transformation, solve the intial value problem.

$$y'' + 9y = 0 \quad , \quad y(0) = 0, y'(0) = 2$$

Table　　Some Elementary Functions $f(t)$ and Their Laplace Transforms $\mathscr{L}(f)$.

	$f(t)$	$\mathscr{L}(f)$		$f(t)$	$\mathscr{L}(f)$
1	1	$1/s$	6	e^{at}	$\dfrac{1}{s-a}$
2	t	$1/s^2$	7	$\cos \omega t$	$\dfrac{s}{s^2 + \omega^2}$
3	t^2	$2!/s^3$	8	$\sin \omega t$	$\dfrac{\omega}{s^2 + \omega^2}$
4	t^n $(n = 1, 2, \cdots)$	$\dfrac{n!}{s^{n+1}}$	9	$\cosh at$	$\dfrac{s}{s^2 - a^2}$
5	t^a (a positive)	$\dfrac{\Gamma(a+1)}{s^{a+1}}$	10	$\sinh at$	$\dfrac{a}{s^2 - a^2}$

(20%) 5. Find the eigenvalues and eigenvectors of the matrix.

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$$