

本試題是否可以使用計算機： 可使用， 不可使用 (請命題老師勾選)

(20%)1. Solve the initial value problem

$$y' = \frac{y}{x} + \frac{2x^3 \cos x^2}{y}, \quad y(\sqrt{\pi}) = 0$$

(20%)2. Find the general solution

$$x^2 y'' - 3xy' + 4y = 0$$

(Hint : $x^2 y'' + axy' + by = 0 \dots$ Cauchy equation or Euler equation)

(20%)3. Find the general solution

$$y^{(5)} - 3y^{(4)} + 3y''' - y'' = 0$$

(20%)4. Solve the nonhomogeneous equation

$$y'' + 4y' = 12$$

(20%)5. Let $f(t) = t \sin \omega t$, Find $\mathcal{L}(f)$

(Hint : $\mathcal{L}(f'') = s^2 \mathcal{L}(f) - sf(0) - f'(0)$)

Table
Some Functions $f(t)$ and Their Laplace Transforms $\mathcal{L}(f)$

	$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$	6	e^{at}	$\frac{1}{s-a}$
2	t	$1/s^2$	7	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
3	t^2	$2!/s^3$	8	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
4	t^n ($n = 1, 2, \dots$)	$\frac{n!}{s^{n+1}}$	9	$\cosh at$	$\frac{s}{s^2 - a^2}$
5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$	10	$\sinh at$	$\frac{a}{s^2 - a^2}$