系所組別： 測量及空間資訊學系

考試科目： 工程數學

考試日期：0219，節次：3

※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. Find an integrating factor and solve the initial value problem. (10%)
$(e^{x+y} + ye^{y})dx + (xe^{y} - 1)dy = 0,\ y(0) = -1$

2. Solve the initial value problem. (10%)
$y'' + y = 0.001x^2,\ y(0) = 0,\ y'(0) = 1.5$

3. Solve the initial value problem. (10%)
$y'' + 2y' + 5y = e^{0.5x} + 40\cos 10x - 190\sin 10x,\ y(0) = 0.16,\ y'(0) = 40.08$

4. Solve the following ODE by power series method. (10%)
$y' = 2xy$

5. Solve the initial value problem by Laplace transform method. (10%)
$y'' + y' + 9y = 0,\ y(0) = 0.16,\ y'(0) = 0$

6. The Fourier series of a function $f(x)$ with period 2π is given by (10%)

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx, \quad n = 1, 2, \cdots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx, \quad n = 1, 2, \cdots$$

Find the Fourier series of the function:

$$f(x) = \begin{cases} -k & , -\pi < x < 0 \\ k & , 0 < x < \pi \end{cases} \quad \text{and} \quad f(x + 2\pi) = f(x)$$

7. Find the rank of the matrix and a basis for the row space. (10 %)

$$\begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & -5 \\ -4 & 5 & 0 \end{bmatrix}$$

(背面仍有題目,請繼續作答)

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8. Find the inverse of matrix A. (10%)

$$A=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix}$$

9. Find the eigenvalues and eigenvectors of matrix B. (10%)

$$B=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

10. Are the following sets of vectors linearly independent? (show the detaiis) (10%)

(a) [3 -2 0 4], [5 0 0 1], [-6 1 0 1], [2 0 0 3]

(b) [1 1 0], [1 0 0], [1 1 1]