國立成功大學 105 學年度碩士班招生考試試題

系　所：測量及空間資訊學系

考試科目：工程數學

考試日期：0227，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. L and L^{-1} are the Laplace and inverse Laplace transformation, respectively. If $L^{-1}[F(s)] = f(t)$ and $L^{-1}[G(s)] = g(t)$, (a) prove that $L^{-1}[F(s)G(s)] = \int_0^t f(t-\tau)\, g(\tau) d\tau$. (15%). (b) Explain the meaning name of this operation and an example of the application. (5%)

2. Solve the following ordinary differential equation with given initial value:

$$y'(x) + 5y(x) = 3x + 7, y(0) = 1$$

(a) Find y(x) by using $y = e^{\lambda x}$. (5%)
(b) Use Laplace transformation to find y(x) and to testify solution from (a). (5%)
(c) Find solution by using power series solutions with center at 0. (5%)

3. The Fourier series of a function f(x) with period 2π is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx,$$

where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx,\ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx,\quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx,$$

(a) Find the Fourier series of the function: (10%)
$f(x) = |x|, -\pi < x < \pi$ and $f(x+2\pi) = f(x)$, for all x
(b) Find the sum of $1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots$.(10%)

4. Given the matrix:

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

Find a matrix S such that $S^{-1}AS = B$, where **B** is a diagonal matrix with eigenvalues of **A** on the diagonal. (20%)

5. (a) Find a least-squares solution of **Ax=b** for (10%)

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} and\ \boldsymbol{b} = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$$

(b) Explain the geometric meaning of least-squares solutions (5%)

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6. Given a scalar function **u**(x,y,z)=zy+yx and a vector function **v**(x,y,z)=[y,z,4z-x]. find: (10%)

(a) $\nabla\mathbf{u}$

(b) $\nabla\cdot\mathbf{v}$

(c) $\nabla\times\mathbf{v}$

(d) $\nabla^2\mathbf{u}$

(e) $\nabla\times(\nabla\times\mathbf{v})$, where ∇ is the gradient operator and ∇^2 is the Laplace operator