國立成功大學 106 學年度碩士班招生考試試題

系　所：測量及空間資訊學系

考試科目：工程數學

考試日期：0213，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let f(x) be a periodic function of period 2L. Prove that f(x) can be represented by the trigonometric series: (15%)

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

Where $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$,

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, \text{n} = 1,2,3, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx, \text{n} = 1,2,3, \ldots$$

2. Find the Fourier series of the function (10%)

$$f(x) = \begin{cases} 0, if -L < t < 0 \\ \text{Esin}\omega\text{t, if } 0 < t < L \end{cases}, L = \frac{\pi}{\omega}$$

3. Solve the initial value problem. (10%)

$$y'' + y' = 0.002x^2, y(0) = 0, y'(0) = 1.5$$

4. For a function u(x,y), if the total differential du is equal to Mdx+Ndy=0, then this function is exactness when $M = \frac{\partial u}{\partial x}, N = \frac{\partial u}{\partial y}$. Using exactness of the following differential equation to solve the equation. (15%)

$$(e^x - \text{siny})dx + (\text{cosy})dy = 0$$

5. Consider the non-honomogenous Euler-Cauchy equation (10%), Find the homogenous solutions

$$x^3y''' - 3x^2y'' + 6xy - 6y = 0$$

6. Find the f(x) by using inverse of the Laplace transform: (15%)

$$L(s) = \frac{3s - 137}{s^2 + 2s + 401}$$

7. Let $\vec{u} = [u_1, u_2, u_3]$ and $\vec{v} = [v_1, v_2, v_3]$, be two vectors. The inner product (or dot product) $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$. Please derive this equation from $\vec{u} \cdot \vec{v} = |\vec{u}| \times |\vec{v}|cos\theta$. (10%)

8. Find the eigenvalues and eigenvectors of matrix B. (15%)

$$B = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$