國立成功大學

110學年度碩士班招生考試試題

編　號： 149

系　所： 測量及空間資訊學系

科　目： 線性代數

日　期： 0202

節　次： 第 2 節

備　註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 5 \\ 4 & 1 & s \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ t \end{bmatrix}$ to be a linear system $\mathbf{Ax} = \mathbf{b}$. (15 %)

(a) If the matrix $\mathbf{A}$ is not invertible, what is the value of s?

(b) Apply the value of s obtained in (a) to this linear system and find the value of t that makes the linear system $\mathbf{Ax} = \mathbf{b}$ have solutions.

2. Let $T: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be a linear transformation defined by (15%)

$$\mathrm{T}\big((x_1, x_2, x_3)\big) = (x_1 - x_3,\ -2x_1 + 3x_2 - x_3,\ 3x_1 - 3x_2).$$

(a) Find the standard matrix **A** for this linear transformation T.

(b) Find a basis for the column space of **A**.

(c) Find a basis for the nullspace of **A**.

3. Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 6 & 6 & 3 \\ -1 & 0 & 0 & 3 \\ 1 & 2 & -2 & 3 \end{bmatrix}$ (15%)

(a) Find the LU decomposition for the matrix **A**, where **L** is a lower triangular matrix with 1 on its diagonal entries and **U** is an upper triangular matrix.

(b) Explain the purpose of LU decomposition.

4. Let $\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4a+5d+6g & 4b+5e+6h & 4c+5f+6i \\ 2a+3d & 2b+3e & 2c+3f \\ a & b & c \end{bmatrix}$.

If the determinant of the matrix **A** is 5, what is the determinant of the matrix **B**? (10%)

5. (a) Find the orthogonal projection of the vector **b** onto the column space of matrix **A**, (b) find the least-squares solution of the system **Ax**=**b**, (c) explain the geometric meaning of the least-squares solution. (15%)

$$\mathbf{A} = \begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$$

6. Diagonalize the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$, if possible. (10%)

7. Given a matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 2 & 1 \end{bmatrix}$, find the singular value decomposition (SVD) of matrix **A**. (20%)