國立成功大學

111學年度碩士班招生考試試題

編　號：150

系　所：測量及空間資訊學系

科　目：線性代數

日　期：0219

節　次：第 2 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) (a) Please prove $det(A^{-1}) = 1/det(A)$. (b) If $A, B, C \in \boldsymbol{R}^{3\times 3}$, C is obtained from A by dividing a row of A by a scalar $\kappa = 2$, $det(A) = 2$, and $det(B) = -1$, then $det\begin{bmatrix} 2I_3 & A \\ 0 & CB \end{bmatrix}$ =?

2. (15%) Find a sigular value decomposition of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$.

3. (15%) Consider the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(a) Is this matrix positive definite, negative definite, or indefinite?

(b) Is this matrix positive definite, negative definite, or indefinite on the subspace $M = \{\boldsymbol{x}: x_1 + x_2 + x_3 = 0\}$?

(c) Consider the quadratic form $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 5x_3^2 + 2\beta x_1x_2 - 2x_1x_3 + 4x_2x_3$. Find th evalues of the parameter β for which this quadratic form is positive definite.

4. (15%) For the linear transformation $T: \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^2$ with $T(a_1, a_2, a_3) = (a_1 - 2a_2, 3a_3)$, find a basis for the nullspace of T, and compute the nullity and rank of T.

5. (20%) Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $\boldsymbol{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $\boldsymbol{c} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$, and define a transformation $T: \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^3$ by $T(\boldsymbol{x}) = A\boldsymbol{x}$, so that $T(\boldsymbol{x}) = A\boldsymbol{x} = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

(a) Find $T(\boldsymbol{u})$, the image of $\boldsymbol{u}$ under the transformation T.

(b) Find an $\boldsymbol{x}$ in $\boldsymbol{R}^2$ whose image under T is $\boldsymbol{b}$.

(c) Is there more than one $\boldsymbol{x}$ whose image under T is $\boldsymbol{b}$?

(d) Determine if $\boldsymbol{c}$ is in the range of the transformation T.

6. (20%) Data points' xy-corrdinate are (0,0), (1,2), (2,4), and (3,6), describing the least-squares line of the equation: $y = \alpha + \beta x$.

(a) Given the design matrix, the observation vector, and the unknow parameter vector(wirte the system as $A\boldsymbol{z} = \boldsymbol{b}$).

(b) Find the unknown parameters that best fit the data points, and conpute the associated least-square error.

(c) Explain the geometric meaning of the least-squares solution.

(d) When $\boldsymbol{b}$ is orthogonal to the columns of A, what can you say about the least-square solution of $A\boldsymbol{z} = \boldsymbol{b}$?