國立成功大學

112學年度碩士班招生考試試題

編　號：151

系　所：測量及空間資訊學系

科　目：線性代數

日　期：0206

節　次：第 2 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the following system of linear equations using inverse matrix method (i.e., $\mathbf{Ax} = \mathbf{b} \Rightarrow \mathbf{x} = \mathbf{A^{-1}b}$). (15%)

$$3x_1 + x_2 + x_3 + 2x_4 = 1$$
$$x_1 + 3x_2 + 2x_3 + 3x_4 = 4$$
$$2x_1 + x_2 + 3x_3 + 3x_4 = 3$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 2$$

2. Calculate the determinant of the matrix $\mathbf{A} = \begin{vmatrix} 0 & 1 & 2 & -1 \\ 2 & 5 & -7 & 3 \\ 0 & 3 & 6 & 2 \\ -2 & -5 & 4 & -2 \end{vmatrix}$. (10%)

3. For the linear system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 9 & 10 & 11 & 12 \\ 16 & 15 & 14 & 13 \end{bmatrix}$,

    answer the following questions.

    (a) Find a basis for the null space $\mathbf{A}$ and the dimension of null space $\mathbf{A}$. (5%)

    (b) Find the column space of $\mathbf{A}$ and the rank of $\mathbf{A}$. (5%)

    (c) Explain how to use matrix ranks in linear equation systems. (10%)

4. Suppose that $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is a basis for a subspace $\mathbf{W}$ in R^4. Generate an orthogonal basis for $\mathbf{W}$. (15%)

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

5. Orthogonally diagonalize the matrix $\mathbf{A} = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, if possible. (20%)

6. Find a singular value decomposition for the matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$. (20%)