

# 國立成功大學

## 113學年度碩士班招生考試試題

編 號：146

系 所：測量及空間資訊學系

科 目：線性代數

日 期：0201

節 次：第 2 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. One of the eigenvalues of the matrix  $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$  is 3. Find a basis for its corresponding eigenspace. (15%)

2. Find a singular value decomposition of the matrix  $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$ . (15%)

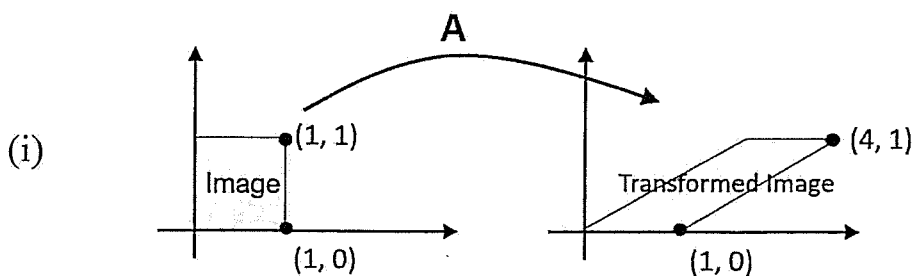
3. Given a linear transformation  $T$ , where (15%)

$$T\left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 13 \\ 14 \\ 15 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- (a) Find the standard transformation matrix  $A$  for  $T$ .  
 (b) Find a basis for the  $\text{Nul } A$  and the nullity (dimension of  $\text{Nul } A$ ).  
 (c) Find a basis for the  $\text{Col } A$  and rank (dimension of  $\text{Col } A$ ).

4. Linear transformation in homogenous coordinate (20%)

- (a) Find the transformation matrices in homogenous coordinate for the following transformations (i) and (ii).



(ii)  $T(\mathbf{x}) = \mathbf{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

- (b) Show that if the transformation  $T(\mathbf{x}) = \mathbf{x} + \mathbf{t}$ , where  $\mathbf{x}, \mathbf{t} \in \mathbb{R}^2$  is a linear transformation?

5. Least-squares linear equation system (20%)

- a). Given a least-squares linear system  $A\mathbf{x} = \mathbf{b}$ . What is the geometric meaning of its

solution  $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$ ?

b). For an ill-conditioned least-squares linear system. What is the difference between the solvers of QR decomposition, singular value decomposition, and  $(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$ .

6. Let  $T: \mathbf{R}^3 \rightarrow \mathbf{R}^3$  be the linear transformation defined by (15%)

$$T(\mathbf{x}) = (x_1 - x_3, -2x_1 + 3x_2 - x_3, 3x_1 - 3x_2).$$

(a) Find the standard matrix  $\mathbf{A}$  for the linear operator  $T$ .

(b) Find the LU-decomposition of matrix  $\mathbf{A}$ , where  $\mathbf{L}$  is a lower triangular matrix and  $\mathbf{U}$  is an upper triangular matrix.

(c) What is the main purpose of LU matrix decomposition?