請注意：本試題禁止使用任何計算器

1. Why is it important to estimate the a posteriori standard deviation of unit weight in adjustment computations? (10%)

2. Please state the relationship between the position precision and the ratio of misclosure in traverse computations. (10%)

3. How do you evaluate the quality of a surveying network? (10%)

4. Assume that the random quantities $X_1, X_2, \cdots, X_n$ are measured two times. The observations are uncorrelated and listed as follows:

$$L_1^{(1)}, L_2^{(1)}, \cdots, L_n^{(1)}$$
$$L_1^{(2)}, L_2^{(2)}, \cdots, L_n^{(2)}$$

with weights: $P_1,\ P_2, \cdots,\ P_3$

The weights of each observation pair ($L_i^{(1)}$ and $L_i^{(2)}$) are supposed to be the same. Compute that

(a) the standard deviation of unit weight, and (10%)

(b) the standard deviation of the i-th mean. (10%)

5. The generalized Gauss-Markov model with full rank is given by

$$\underset{n\times1}{\ell} + \underset{n\times1}{v} = \underset{n\times u}{A}\ \underset{u\times1}{x}, \qquad \Sigma_{\ell\ell} = \sigma_0^2 P^{-1} \tag{1}$$

If the least squares principle is applied, prove that

(a) $\Sigma_{\hat{v}\hat{v}} = \sigma_0^2 Q_{\hat{v}\hat{v}} = P^{-1} - A(A^T PA)^{-1} A^T$ (10%)

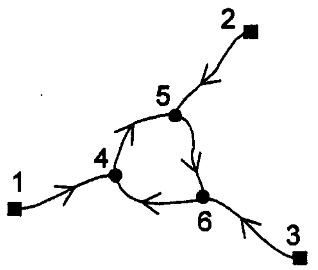
(b) $\hat{v} = -Q_{\hat{v}\hat{v}} P\,\ell$ (5%)

(背面仍有題目,請繼續作答)

6. If the parameter x in the generalized Gauss-Markov model (1) is subject to the constraint $Cx = c_0$, where C denotes a $d \times u$ matrix of known coefficients with rank $(C) = d \leq u$ and c_0 is a known $d \times 1$ vector. If the least squares principle is applied, prove that the estimator $\hat{x}$ of the vector x is given by (15%)

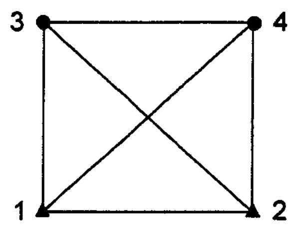
$$\hat{x} = (A^T PA)^{-1}[A^T P\ell + C^T (C(A^T PA)^{-1} C^T)^{-1}(c_0 - C(A^T PA)^{-1} A^T P\ell)]$$

7. Give you the observation equations without solving them by any adjustment principles for each network sketched in the following. Note that the known points denoted by the symbol ■ or ▲ are the weighted points. The symbol Δ means the difference, for example $\Delta H_{ij} = H_j - H_i$ and $\Delta X_{ij} = X_j - X_i$. (20%)



Given Heights: H_1, H_2, H_3
with weights: W_1, W_2, W_3
Observations: ΔH_{14}, ΔH_{25}, ΔH_{36}, ΔH_{45}, ΔH_{56}, ΔH_{64}
with weights: P_1, P_2, P_3, P_4, P_5, P_6
Found: H4, H5, H6

Leveling network



Given : X_1, Y_1, Z_1
X_2, Y_2, Z_2
with weights: W_{X1}, W_{Y1}, W_{Z1}
W_{X2}, W_{Y2}, W_{Z2}
Observations: ΔX_{ij}, ΔY_{ij}, ΔZ_{ij}
i,j=1,2,3,4, $i \neq j$
with weights: P_{ij}, i,j=1,2,3,4, $i \neq j$
Found: X_3, Y_3, Z_3, X_4, Y_4, Z_4

GPS baseline network