系所組別： 測量及空間資訊學系
考試科目： 工程數學
考試日期：0307，節次：3

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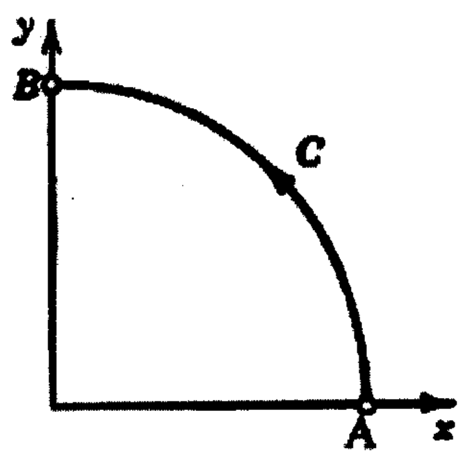
1. (a) Please prove the definite integral $\int_a^b f(x)dx = F(b) - F(a)$ holds, if the integrand $f(x)$ is equal to the derivative $F'(x)$ of a continuous function $F(x)$. (10%)

   (b) Let $f(x)$ be a continuous and integrable function of a real variable x. Its **Fourier Transform** is defined by $F(u) = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx$ with $j=\sqrt{-1}$. If $f(x) = \begin{cases} 2 & ,x\in[0,4] \\ 0 & otherwise \end{cases}$, the Fourier Transform of f and its Fourier spectrum are $F(u) = \frac{b}{\pi u}\sin(\pi au)e^{-j\pi au}$ and $|F(u)| = c\left|\frac{\sin(\pi au)}{(\pi au)}\right|$, respectively. Please determine the values of a, b and c. (10%)

2. A **line integral** of a vector function $\vec{F}(\vec{r})$ over a curve C is defined by $\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}dt$. In terms of components, with $d\vec{r} = [dx, dy, dz]$ and $' = d/dt$, it becomes $\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \int_C (F_1dx + F_2dy + F_3dz) = \int_a^b (F_1x' + F_2y' + F_3z')dt$.
   Please find the value of the line integral $\int_C \vec{F}(\vec{r})\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}dt$ when $\vec{F}(\vec{r})=[-y, -xy]=-y\vec{i} -xy\vec{j}$ and C is the circular arc as in the following figure from A (1,0) to B(0, 1). (20%)



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3. Let $\vec{u}=[u_1 \quad u_2 \quad u_3]$ and $\vec{v}=[v_1 \quad v_2 \quad v_3]$ be two vectors. The **inner product** (or **dot product**) $\vec{u}\cdot\vec{v}$ of these two vectors $\vec{u}$ and $\vec{v}$ can also be computed by $\vec{u}\cdot\vec{v}=u_1v_1+u_2v_2+u_3v_3$. Please derive this equation $\vec{u}\cdot\vec{v}=u_1v_1+u_2v_2+u_3v_3$. (15%)

4. Solve $x^3\dfrac{d^3y}{dx^3}+5x^2\dfrac{d^2y}{dx^2}+7x\dfrac{dy}{dx}+8y=0$. (15%)

5. L and L^{-1} are the Laplace and inverse Laplace transformation, respectively. If $L^{-1}[F(s)]=f(t)$ and $L^{-1}[G(s)]=g(t)$, prove that $L^{-1}[F(s)G(s)]=\int_0^t f(t-\alpha)g(\alpha)d\alpha$. (15%)

6. Solve the initial value problem $y''+w^2y=F_0\cos wx$, with $y(0)=y'(0)=0$, where F_0 and w are constant. (15%)