系所組別： 測量及空間資訊學系

考試科目 工程數學

考試日期：0307，節次：3

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1. Solve the following initial value problems. (16%)

   (a) $y \ + 2xy = 0,\ y = ce^{-x^2},\ y(1) = 1/e$

   (b) $y \ - 2y \ - 3y = 0,\ y(0) = 2,\ y\ (0) = 14$

   (c) $x^2 y \ - 4xy \ + 6y = 0,\ y(1) = 1,\ y'(1) = 0$

   (d) $y'' + 4y = 16\cos 2x,\ y(0) = 0,\ y'(0) = 0$

2. Find the general and particular solution of the following differential equation: (10%)

   $y \ + 4y' + 4y = 4x^2 + 6e^x$

3. (a) Test the exactness of the following differential equation: (10%)

   $(e^x - \sin y)dx + (\cos y)dy = 0$

   (b) With the help of the finding from part (a), solve this differential equation.

4. Construct a 2nd order linear non-homogenous ODE such that its homogenous solution are e^{2x} and xe^{2x}, and its particular solution is $x + 1 + e^x$ (10%)

5. Consider the non-homogenous Euler-Cauchy equation (10%)

   $x^3 y''' - 3x^2 y'' + 6xy \ - 6y = 0$

(a) Find the homogenous solutions (Hint: the roots of $m^3 - 6m^2 + 11m - 6 = 0$ are 1, 2, and 3)

(b) Use the method of variation of parameters to find the particular solution.

6. The Fourier series of a function $f(x)$ with period $2L$ is given by (16%)

$$a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$$

where

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, \quad n = 1, 2, \cdots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx, \quad n = 1, 2, \cdots$$

Find the Fourier series of the functions:

(a) $f(x) = e^x,\ -\pi < x < \pi$

(b) $f(x) = \begin{cases} 0 \ , -\pi < x < 0 \\ x^2, 0 < x < \pi \end{cases}$

(背面仍有題目,請繼續作答)

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7. If $A = \begin{bmatrix} 4 & 2 & 5 \\ -1 & 0 & 5 \\ 2 & 1 & 2 \end{bmatrix}$, then (answer 'yes' or 'no') (10%)

(A) rank(A)=2.

(B) det(A)=1.

(C) $\begin{bmatrix} 2 & 1 & 2 \end{bmatrix}^T$ is in the nullspace of A.

(D) If $b = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, then the linear system $Ax = b$ is consistent.

(E) A is row equivalent to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8. Which L is a linear transformation? (5%)

(A) $L(X) = X + \begin{bmatrix} 1 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, X \in R^{2x4}$

(B) $L(A) = A - A^T, \quad A \in R^{nxn}$

(C) $L(\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T) = x_1 + \sqrt{2}x_2 - x_3$.

(D) $L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ 1 \end{bmatrix}$..

(E) None of the above is linear.

9. Given the matrix (13%)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

(a) Find a matrix S such that $S^{-1}AS = B$, where B is a diagonal matrix with eigenvalues of A on the diagonal.

(b) Calculate e^A