編號： 87

國立成功大學104學年度碩士班招生考試試題

系所組別：資源工程學系甲組

考試科目：工程數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. How many degrees are $\left(\frac{d^4y}{dx^4}\right)^3 - \frac{3}{2}e^x\frac{d^2y}{dx^2} - y + x = 0$? (5%)

2. Is $\left(\frac{d^4y}{dx^4}\right)^3 - \frac{3}{2}e^x\frac{d^2y}{dx^2} - y + x = 0$ homogeneous? (5%)

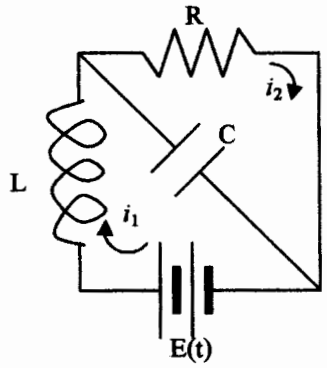
3. Is $\left(\frac{d^4y}{dx^4}\right)^3 - \frac{3}{2}e^x\frac{d^2y}{dx^2} - y + x = 0$ linear? (5%)

4. Is $(x^2 + y^2)dx - 2xydy = 0$ exact? (5%)

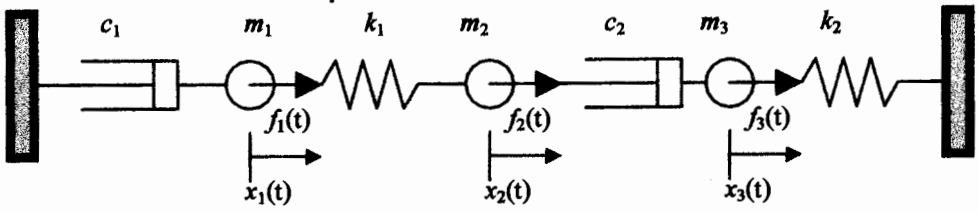
5. Find an integrating factor to make $(e^{x+y} + ye^y)dx + (xe^y - 1)dy = 0$ exact. (5%)

6. Under what conditions for the constants a, b, c, and e to make $(ax + by)dx + (cx + ey)dy = 0$ exact. (5%)

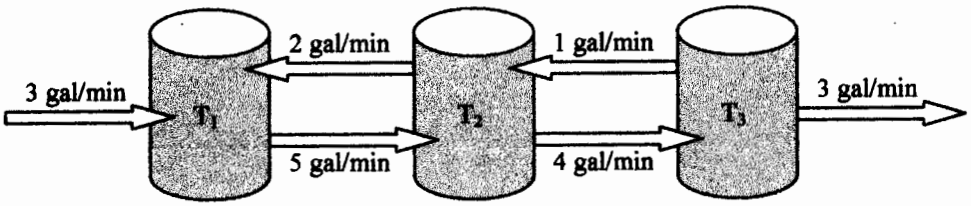
7. Write down the governing equations for the currents: i_1 and i_2? (5%)



8. Write down the governing equations in terms of $x_1(t)$, $x_2(t)$, $x_3(t)$? (Don't solve) (5%)



9. Tank 1 initially contains 100 gal of water in which 100 lb of salt are dissolved. Tank 2 and Tank 3 initially each contain 100 gal of pure water. The inflow to Tank 1 is 3 gal/min containing 3 lb of salt from outside. The connection between these tanks are shown in the following figure. Write down the governing equations and initial conditions for the salt content $y_1(t)$, $y_2(t)$, and $y_3(t)$ in Tank 1, 2, and 3, respectively. (Don't solve) (5%)



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10. Is x=0 ordinary, regular singular or irregular singular for $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-1)y=0$? (5%)

11. Calculate the Laplace transforms of $\sin(at+b)$? (5%)

12. Find the inverse Laplace transform of $\frac{2}{s^2+1}e^{-3s}$? (5%)

13. Use the Laplace transform to solve the following problems: $y(t)=e^{-t}+\int_0^t y(t-\tau)d\tau$? (5%)

14. Find the reduced row echelon form of a matrix $\begin{bmatrix}1&0&0&0\\0&2&1&1\\0&1&2&1\\0&1&1&2\end{bmatrix}$ and its rank? (5%)

15. Find the row space of $\begin{bmatrix}1&0&0&0\\0&2&1&1\\0&1&2&1\\0&1&1&2\end{bmatrix}$ and its dimension? (5%)

16. Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix}2&0&0\\0&2&1\\0&1&2\end{bmatrix}$? (5%)

17. Determine the Fourier series expansion of the periodic function: $f(x)=\begin{cases}0, & -\pi<x<0\\ x^2, & 0<x<\pi\end{cases}$ with fundamental period 2π? (5%)

18. Determine the Fourier cosine integrals of the function: $f(t)=e^{-t}, \quad t>0$? (5%)

19. Find the inverse Fourier transform of the function $\frac{1}{3+4i\omega-\omega^2}$? (5%)

20. An equation is given as: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ with boundary conditions: $u(0,t)=0$ & $u(L,t)=0$ for $\forall t\geq 0$ and initial conditions: $u(x,0)=\sin\left(\frac{\pi x}{L}\right)$ for $0\leq x\leq L$. The solution of above system is: $u(x,t)=\sum_{n=1}^{\infty}B_n e^{-\lambda_n^2 t}\sin(\frac{n\pi}{L}x)$ in which $\lambda_n=\frac{cn\pi}{L}$ where $n=1,2,3,\cdots$. Evaluate B_n? (Carry out the integration) (5%)