

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. How many orders are $\left(\frac{dy}{dx}\right)^3 - \frac{3}{2}e^x \frac{d^2y}{dx^2} - y + x = 0$? (4%)

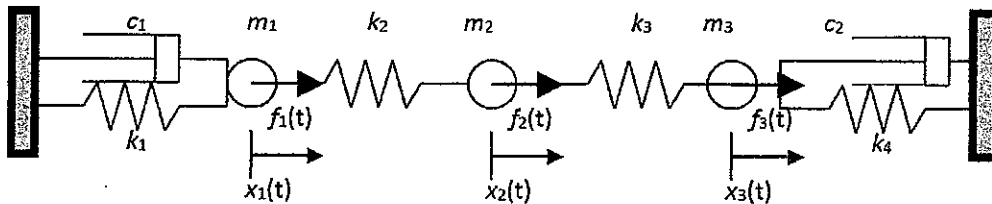
2. Is $\left(\frac{dy}{dx}\right)^3 - \frac{3}{2} \frac{d^2y}{dx^2} + \sin(x) = 0$ homogeneous? (4%)

3. Is $\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} - x^2 \frac{\partial u}{\partial y} - e^x = 0$ linear? (4%)

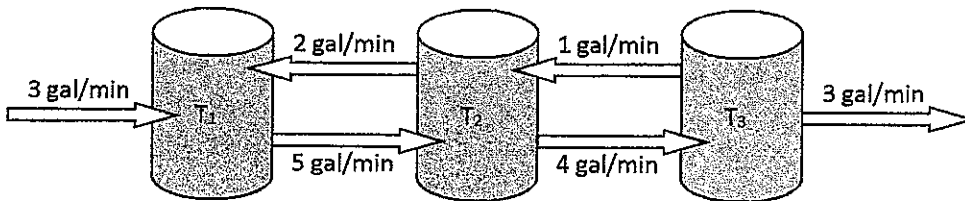
4. Is $\cos(x+y)dx - 2xydy = 0$ exact? Why? (4%)

5. Find an integrating factor to make $-ydx + xdy = 0$ exact. (4%)

6. Write down the governing equations in terms of $x_1(t), x_2(t), x_3(t)$? (Don't solve) (5%)



7. Tank 1 initially contains 100 gal of water in which 100 lb of salt are dissolved. Tank 2 initially contain 200 gal of water in which 200 lb of salt are dissolved. Tank 3 initially contains 300 gal of pure water. The inflow to Tank 1 is 3 gal/min containing 3 lb of salt from outside. The connection between these tanks are shown in the following figure. Write down the governing equations and initial conditions for the salt content $y_1(t), y_2(t),$ and $y_3(t)$ in Tank 1, 2, and 3, respectively. (Don't solve) (5%)



8. Write down the indicial equation and its roots for $x^2 \frac{d^2y}{dx^2} + (3x^2) \frac{dy}{dx} + 1.5y = 0$ at $x=0$? (4%)

9. Calculate the Laplace transforms of $\cosh(at + b)$? (4%)

10. Find the inverse Laplace transform of $\frac{1}{s^2 + 4} e^{-7s}$? (4%)

11. Use the Laplace transform to solve the following problems: $y''(t) + y(t) = \delta(t-1), y(0) = 0, y'(0) = 3$ in which

$\delta(t-1) = \begin{cases} \infty, t = 1 \\ 0, t \neq 1, t > 0 \end{cases}$ is a Dirac delta function? (5%)

12. Find the reduced row echelon form of a matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ and its rank? (5%)

13. Find the null space of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ and nullity? (5%)

14. Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$? (5%)

15. Find the (a)divergence) (b)curl of $yz\vec{i} + 3xz\vec{j} + z\vec{k}$ at $(1, 1, 1)$? (6%)

16. Find the direction in which $\sin(x+2y+3z)$ has a maximum change of rate in position at $(2\pi, -\pi, 0)$? (4%)

17. Determine the Fourier series expansion of the periodic function: $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ with fundamental period 2π ? (5%)

18. Determine the Fourier sine integrals of the function: $f(t) = e^{-t}, t > 0$? (5%)

19. Find the inverse Fourier transform of the function $\frac{1}{1+\omega^2}$? (5%)

20. An equation is given as: $\frac{\partial u(x,y)}{\partial x \partial y} = 0, x \geq 0, y \geq 0$ with boundary conditions: $u(x, 0) = x(1-x), u(0, y) = y, u(1, y) = y$.

Find the solution of above system? (5%)

21. Classify following 2nd order partial differential equation: $t \frac{\partial^2 u(x,t)}{\partial x^2} + x \frac{\partial^2 u(x,t)}{\partial x \partial t} + \frac{\partial^2 u(x,t)}{\partial t^2} = 0$? (4%)

22. Find b and c to make the given function: $x^3 + bx^2y + cxy^2 + y^3$ a harmonic function? (4%)