

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Is $(\cos x \sin y)dx + (\sin x \cos y)dy = 0$ exact? Why? (5%)

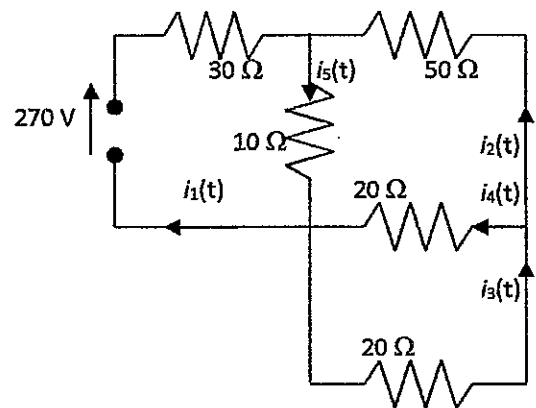
2. Find an integrating factor F to make $dx + (3x - e^{-2y})dy = 0$ exact. (5%)

3. Solve $x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 9y = 0$, $x > 1$, $y(1) = 1$, $y'(1) = 0$? (5%)

4. Solve $y'' + y = \sin x$, $y(0) = 0$, $y'(0) = 1$? (5%)

5. A tank contains V gal of brine water in which y_0 lb of salt is dissolved. Fresh water runs into the tank at a rate of w gal/min. The mixture, kept uniform by stirring, runs out at a rate of w gal/min. Assume that the amount of salt at time t minutes is $y(t)$. Write down the differential equation governing the salt, $y(t)$, in the tank and solve for $y(t)$? Express your answer in terms of t , y_0 , V and w . (5%)

6. Find the currents: $i_1(t)$, $i_2(t)$, $i_3(t)$, $i_4(t)$, $i_5(t)$? (5%)



7. $y(t) = 1 + \int_0^t y(t-\tau) d\tau$ (5%)

8. Calculate the Laplace transforms of $\sin(at+b)$ where a and b are constants? (5%)

9. Find the inverse Laplace transform of $\frac{s}{s^2-4} e^{-s}$? (5%)

10. Use the Laplace transform to solve the following problems: $y''(t) + y(t) = u(t-1)$, $y(0) = 3$, $y'(0) = 0$ in which

$$u(t-1) = \begin{cases} 0, & t < 1 \\ 1, & t > 1 \end{cases} \text{ is a unit step function? (5\%)}$$

11. Find the inverse of a matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 1 & 3 & 1 & 4 \end{bmatrix}$ and its rank? (5%)

12. Find the null space of $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 0 & 1 & 1 & 1 \\ 2 & -2 & 0 & 0 \\ 1 & 0 & 1 & -4 \end{bmatrix}$ and its nullity? (5%)

13. Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$? (5%)

14. Find the (a)divergence) (b)curl of $yz^2\vec{i} + 3xz\vec{j} + 5z\vec{k}$ at $(1, 2, 3)$? (10%)

15. Find the direction in which $\cos(x+3y+2z)$ has a maximum change of rate in position at $(2\pi, 0, -\pi)$? (5%)

16. Determine the Fourier series expansion of the periodic function: $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ with fundamental period 2π ? (5%)

17. Determine the Fourier cosine integrals of the function: $f(t) = e^{-t}, t > 0$? (5%)

18. Find the Fourier transform of the function $f(x) = \begin{cases} 1, & -\pi < x < \pi \\ 0, & \text{elsewhere} \end{cases}$? (5%)

19. An equation is given as: $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $t > 0$, $0 < x < 1$ with boundary conditions: $u(x, 0) = x(1-x)$, $u(0, t) = u(1, t) = 0, t > 0$. Find the solution of above system? (5%)