編號： 90

國立成功大學 107 學年度碩士班招生考試試題

系　　所：資源工程學系

考試科目：工程數學

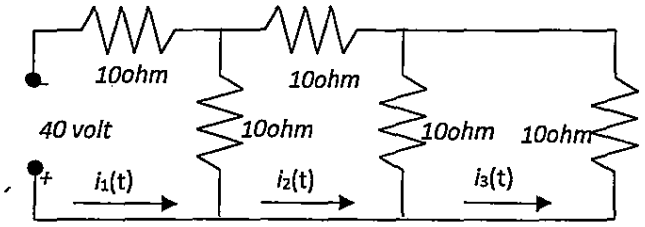
考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. An integrating factor of the form $x^a y^b$ makes $(a+1)ydx+(b+1)xdy=0$ exact. What are a and b? (10%)

2. Solve $y''+4y=\sin 2x$, $y(0)=0$ $y'(0)=1$? (10%)

3. An electrical circuit is shown in the figure. Write down the required equations in terms of the electric currents: $i_1(t)$, $i_2(t)$, and $i_3(t)$ and solve them? (10%)



4. Calculate the Laplace transforms of $\cosh(at+b)$ where a and b are constants? (5%)

5. Find the inverse Laplace transform of $\frac{s^2}{s^2+4}e^{-s}$? (5%)

6. Use the Laplace transform to solve the following problems: $y''(t)+y(t)=\delta(t-1)$, $y(0)=3, y'(0)=0$ in which

$\delta(t-1)=\begin{cases}0, t\neq 1\\ \infty, t=1\end{cases}$ is a unit impulse function(Direc's delta)? (10%)

7. Find the rank, null space, and nullity of the matrix: $\begin{bmatrix}1&0&0&0&0\\0&1&1&1&1\\1&0&1&0&1\\0&1&1&1&0\end{bmatrix}$? (10%)

8. Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix}-2&2&-3\\2&1&-6\\-1&-2&0\end{bmatrix}$? (5%)

9. Find the (a)divergence (b)curl of $z^2\vec{i}+3xyz\vec{j}+5xz\vec{k}$ at (1, 2, 3)? (5%)

10. Find the direction in which $\sin(3x+2y+z)$ has a maximum change of rate in position at $(\pi, 2\pi, -\pi)$? (5%)

11. Determine the Fourier series expansion of the periodic function: $f(x)=\begin{cases}-1, & -\pi<x<0\\ 1, & 0<x<\pi\end{cases}$ with period 2π ? (10%)

12. Determine the Fourier sine integrals of the function: $f(t)=e^{-t}, t>0$? (10%)

13.An equation is given as: $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ with boundary conditions: $u(0,t)=0$ & $u(L,t)=0$ for $\forall t\geq 0$ and initial conditions:

$u(x,0)=\sin\left(\frac{\pi x}{L}\right)+\sin\left(\frac{5\pi x}{L}\right)+\sin\left(\frac{9\pi x}{L}\right)$ for $0\leq x\leq L$. The solution of above system is: $u(x,t)=\sum_{n=1}^{\infty}B_n e^{-\lambda_n^2 t}\sin(\frac{n\pi}{L}x)$ in

which $\lambda_n=\frac{cn\pi}{L}$ where $n=1,2,3,\cdots$. Evaluate B_n ? (5%)