※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Is $\sin(x+y)dx - \cos(x+y)dy = 0$ exact? Why? (7%)

2. Find an integrating factor to make $-ydx + xdy = 0$ exact. (7%)

3. Calculate the Laplace transforms of $\cosh(at+b)$? (7%)

4. Find the inverse Laplace transform of $\dfrac{1}{s^2-4}e^{-7s}$? (7%)

5. Use the Laplace transform to solve the following problems: $y''(t) + 4y(t) = \delta(t-1)$, $y(0) = 0$, $\qquad y'(0) = 0$ in which

$\delta(t-1) = \begin{cases} \infty, t = 1 \\ 0, t \neq 1, t > 0 \end{cases}$ is a Dirac delta function? (7%)

6. Find the reduced row echelon form of a matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ and its rank? (8%)

7. Find the null space of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ and nullity? (8%)

8. Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 5 & 0 \\ -1 & 0 & 3 \end{bmatrix}$? (7%)

9. Find the (a)divergence) (b)curl of $yz\vec{i} + xz\vec{j} + xy\vec{k}$ at (1, 2, 3)? (7%)

10. Find the direction in which $\sin(x+y+z)$ has a maximum change of rate in position at (2π, -π, 0)? (7%)

11. Determine the Fourier series expansion of the periodic function: $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ with fundamental period 2π ?
(7%)

12. Determine the Fourier sine integrals of the function: $f(t) = e^{-t}, t > 0$? (7%)

13. Find the Fourier transform of the function $f(x) = e^{-|x|}$? (7%)

14. Find a and b to make the given function: $x^3 + ax^2y + bxy^2 + y^3$ a harmonic function?(7%)