※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (16%)

(a) If $\dfrac{d^2 y}{dx^2} + y = \sin x$, $\quad y(0) = 0, \dfrac{dy(0)}{dx} = 0$, find $y(x)$?

(b) If $\dfrac{dy}{dx} = x + y + 2$, $\quad y(0) = 1$, find $y(x)$?

2. (16%)

(a) Calculate the Laplace transforms of $\cosh(at + b)$?

(b) Find the inverse Laplace transform of $\dfrac{1}{s^2 - 4} e^{-7s}$?

3. (16%)

(a) Find the reduced row echelon form of a matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ and its rank?

(b) Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$?

4. (16%)

(a) Find the ⓐ divergence and ⓑ curl of. $yz^2\vec{i} + x^2 z\vec{j} + xy^2\vec{k}$ at $(1, 2, 3)$?

(b) Find the direction in which $\cos(x + y + z)$ has a maximum change of rate in position at $(2\pi, -\pi, 0)$?

5. (16%)

(a) Determine the Fourier series expansion of the periodic function: $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ with fundamental period 2π ?

(b) Find the Fourier transform of the function $f(x) = e^{-|x|}$?

6. (20%)

Write a 600-word essay about you and mathematics?