

國立成功大學
110學年度碩士班招生考試試題

編 號： 82

系 所： 資源工程學系

科 目： 工程數學

日 期： 0202

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (16%)

(a) If $\frac{d^2 y}{dx^2} + y = \cos x$, $y(0) = 0$, $y(\pi) = 0$, find $y(x)$, $0 < x < \pi$?

(b) If $\frac{dy}{dx} = y+x$, $y(0) = 1$, find $y(x)$?

2. (16%)

(a) Calculate the Laplace transform of $\sin(at+b)$ where a and b are real constants. ?

(b) Find the inverse Laplace transform of $\frac{1}{s^2+4} e^{-3s}$?

3. (18%)

(a) Decompose following matrix:

$$\begin{bmatrix} 1 & 4 & 2 \\ 4 & 25 & 26 \\ 2 & 26 & 44 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

(b) $[A] = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 3 & 3 & 6 \\ 1 & 4 & 5 & 9 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

(b-1) find the rank of $[A]$?

(b-2) find the trace of $[A]$?

(b-3) find the determinant of $[A]$?

(c) Find the eigenvalues and the corresponding eigenvectors of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$?

4. (15%)

(a) Find the divergence and (b) curl of $x^2 \vec{i} + 3\vec{j} - x\vec{k}$ at $(1, 2, 3)$?

(c) Find the gradient of $xy^2 - 3z^3$ at $(1, -2, 4)$?

5. (20%)

(a) Determine the Fourier series expansion of the periodic function: $f(x) = \begin{cases} 0, & -1 \leq x \leq 0 \\ x, & 0 \leq x \leq 1 \end{cases}$ with a fundamental period 2?

(b) Find the Fourier integral of the function $f(x) = e^{-|x|}$?

6. (15%)

Give an example of applications relating to 1st order ordinary differential equation?