

國立成功大學

111學年度碩士班招生考試試題

編 號：82

系 所：資源工程學系

科 目：工程數學

日 期：0219

節 次：第 3 節

備 註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (18%)

(a) If  $\frac{d^2y}{dx^2} + y = \sin(2x)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ , find  $y(x)$ ,  $x > 0$ ?

(b) If  $\frac{dy}{dx} = y + e^x$ ,  $y(0) = 2$ , find  $y(x)$ ?

2. (12%)

(a) Calculate the Laplace transform of  $e^{3t+2}$ ?

(b) Find the inverse Laplace transform of  $\frac{s+6}{s^2+4s+20}$ ?

3. (18%)

(a)  $[A] = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 1 & -3 \\ 1 & 2 & 3 & 1 \\ 1 & 3 & 0 & 1 \end{bmatrix}$

(a-1) find the row reduced echelon form of  $[A]$ ?

(a-2) find the rank of  $[A]$ ?

(a-3) find the trace of  $[A]$ ?

(b) Find the eigenvalues and the corresponding eigenvectors of  $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ?

4. (18%)

(a) Find the divergence of  $3xy^2\vec{i} - 2yz^2\vec{j} + 4x^2z\vec{k}$  at  $(-1, 2, 3)$ ?

(b) Find the directional derivative of  $x+y^2+z^3$  along the direction of  $2\vec{i}+2\vec{j}+1\vec{k}$  at  $(3, 2, -1)$ ?

(c) State the Green's theorem in a plane?

5. (24%)

(a) Determine the Fourier series expansion of the periodic function:  $f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$  with a fundamental period 2?

(b) Find the Fourier cosine integral of the function  $f(x) = e^{-x}$ ,  $x > 0$ ?

(c) Find the Fourier transform of the function  $f(x) = e^{-|x|}$ ,  $x \in \mathbb{R}$ ?

6. (10%)

Give an example of applications relating to 2<sup>nd</sup> order ordinary differential equation?