

國立成功大學

114學年度碩士班招生考試試題

編 號：64

系 所：資源工程學系

科 目：工程數學

日 期：0211

節 次：第 3 節

注 意：1.不可使用計算機
2.請於答案卷(卡)作答，於
試題上作答，不予計分。

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1. Solve the ordinary differential equations.

(1) $\frac{dy}{dx} = -\frac{4x}{y}$ with $y(2) = 3$ (10%).

(2) Find the general solution and particular solution to $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12x^2$ (10%).

2. Apply the Laplace transform to solve the ordinary differential equation: $\frac{d^2y}{dt^2} + 9y = \delta(t - \frac{\pi}{2})$ with the initial conditions: $y(0) = 2$, $y'(0) = 0$ and the Dirac delta function: $\delta(t - \frac{\pi}{2})$ (20%).

3. (1) Find the gradient for the function $f = \ln(x^2 + y^2)$ with the coordinates (x, y) , and the divergence and curl of this gradient (15%).

(2) Express Laplace's equation in terms of f . Explain whether f satisfies Laplace's equation (5%).

4. (1) Find the rank, nullity, and inverse of the matrix **A** (10%).

(2) Determine the eigenvalues and corresponding eigenvectors of the matrix **B** (10%).

$$\mathbf{A} = \begin{bmatrix} 0 & -0.2 & 0.75 \\ 0.4 & 1 & 2 \\ 0 & 0 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

5. (1) Prove that the Fourier sine transform has the property:

$$F_s \left\{ \frac{d^2 f}{dx^2} \right\} = -w^2 F_s \{ f(x) \} + \sqrt{\frac{2}{\pi}} w f(0)$$

where the transform to function $f(x)$ with $0 \leq x < \infty$ is defined as $F_s \{ f(x) \} =$

$$\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(wx) dx, \text{ and the remote boundary condition: } \lim_{x \rightarrow \infty} f = 0 \text{ (10\%).}$$

(2) Apply this property to explain why the Fourier sine transform cannot be used to solve the ordinary

$$\text{differential equation } \frac{d^2 f}{dx^2} + \frac{df}{dx} + f = 0 \text{ with } f(0) = 1 \text{ and } \lim_{x \rightarrow \infty} f = 0 \text{ (10\%).}$$