國立成功大學

115學年度碩士班招生考試試題

編　號：59

系　所：資源工程學系

科　目：工程數學

日　期：0204

節　次：第 3 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

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1. Solve the ordinary differential equations.

(1) $\frac{dy}{dx} = -\frac{1}{y}\sin x$ with $y(0) = 1$ (10%).

(2) Find the general solution and particular solution to $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x$ (10%).

2. Apply the Laplace transform to solve the ordinary differential equation: $\frac{dy}{dt} + y = \delta(t-1)$ with the initial condition: $y(0) = 1$, and the Dirac delta function: $\delta(t-1)$ (20%).

3. Let $f = xz - yz$, $\mathbf{v} = [4z, 2y, x - z]$, and $\mathbf{w} = [y^2, y^2 - x^2, 2z^2]$ with the coordinates (x, y, z). Find

(1) gradient of f at the location (0, 3, 1) (5%)

(2) divergences of $\mathbf{v}$ and $\mathbf{w}$ (10%)

(3) curl of $\mathbf{v}$ (5%)

4. (1) Find the rank, and nullity of the matrix **A** (10%).

(2) Explain how many solutions the linear system ($\mathbf{A}\,\mathbf{x} = \mathbf{B}$) will have (10%).

$$\mathbf{A} = \begin{bmatrix} 5 & -3 & 1 \\ 2 & 3 & -1 \\ 8 & 9 & -3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$$

5. (1) Apply the integration by parts to prove that the integral transform defined as

$F\{f(x)\} = \int_0^b f(x)\sin(\beta x)\,dx$ has the property:

$$F\left\{\frac{d^2f}{dx^2}\right\} = -\beta^2 F\{f(x)\}$$

with $\beta = n\pi/b$, $n = 1, 2, \ldots, \infty$, and the boundary conditions: $f(0) = 0$ and $f(b) = 0$ (10%).

(2) Apply this property to solve the ordinary differential equation: $\frac{d^2f}{dx^2} + f = 1$ with $f(0) = 0$, $f(b) = 0$, and $b = 1$. The inverse integral transform is defined below (10%).

$$f(x) = \frac{2}{b}\sum_{n=1}^{\infty} F(\beta)\sin(\beta x)$$